Intro to cartography: Reference systems and basic principles of geodesy

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What is cartography dealing with?

A map is nothing but a **flat figure**, which represents the **surface of the Earth** or a portion of it (Lagrange 1736-1813)

By cartography we mean "the complex of scientific, artistic and technical studies and operations that take place starting from the results of **direct observations** or from the use of documentation in order to **elaborate and prepare maps**, plants and other modes of expression suitable for awaken the exact image of reality "(International Cartographic Association 1966)

What is cartography dealing with?

Maps represent the territory showing:

- the planimetric position of the points (x, y)
- in some cases they report the odds
- attributes (symbols, ...)

A map is a scaling of reality

The scale is the ratio between the distance measured on the map and the corresponding distance measured in reality. The scale is often referred to as a fraction with a numerator of 1

Es. 1:1000 → 1 mm map= 1000 mm reality = 1 m reality

What is cartography dealing with?

Some examples:

scale 1 : 5.000	1 cm = 50 m
scale 1 : 10.000	1 cm = 100 m
scale 1 : 15.000	1 cm = 150 m
scale 1 : 25.000	1 cm = 250 m
scale 1 : 50.000	1 cm = 500 m
scale 1 : 200.000	1 cm = 2 km
scale 1 : 1.000.000	1 cm = 10 km

Large scale

Small scale

Graphicism error

It is used to evaluate the metric uncertainty of the points represented on the map

We assume e = 0.2 mm on the map

Through the *m*-scale I will be able to have the corresponding information in terms of the real object

Example:

scale 1:1000

0.2 mm x 1000 = 200 mm = 20 cm

Cartography

A map is the projection of a portion of the territory, its morphology and its natural and anthropic aspects.

→ It must allow to derive the position relations existing on the ground among the details represented on the map.

FUNDAMENTAL CHARACTERISTICS

- Geodetic network used for the survey (trigonometric points)
- Altitude indications (listed points, contour lines)
- Essential topographic references (e.g. communication lines, urbanized areas)
- Representation of surface hydrography (rivers, lakes, springs)

Cartography

According to their construction maps are distinguished:

Surveyed maps - built on the basis of surveys on the ground: the position of the elements is realized with geodetic, topographic and photogrammetric measurements

Derived maps - built from a reduction of one or more existing maps (detected or in turn derived) to a larger scale

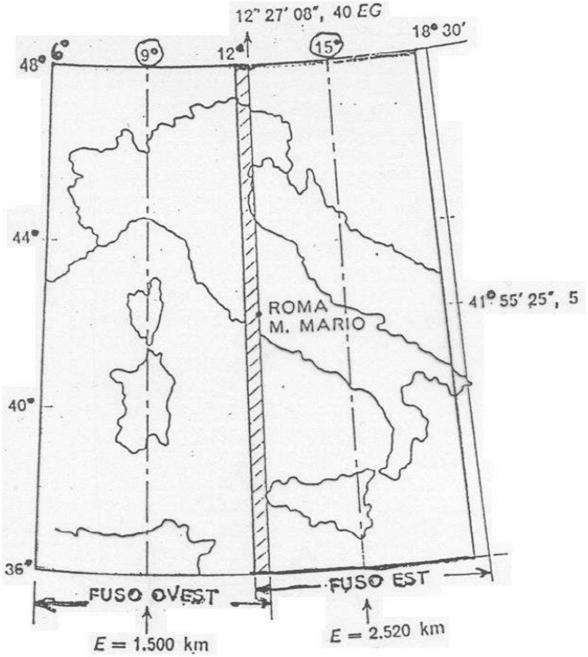
example: official Italian IGM cartography

surveyed maps = 1: 25,000 (tablet)
derived maps = 1: 50,000 (quadrant) and 1: 100,000 (sheet) ca

How can I make a 3D object 2D?

3D

2D



Reference system

It is a set of mathematical rules and measures (angles, distances to the ground, differences in level) that allow to associate points of the physical surface of the Earth with planimetric coordinates (latitude and longitude) and altimetric altitude)

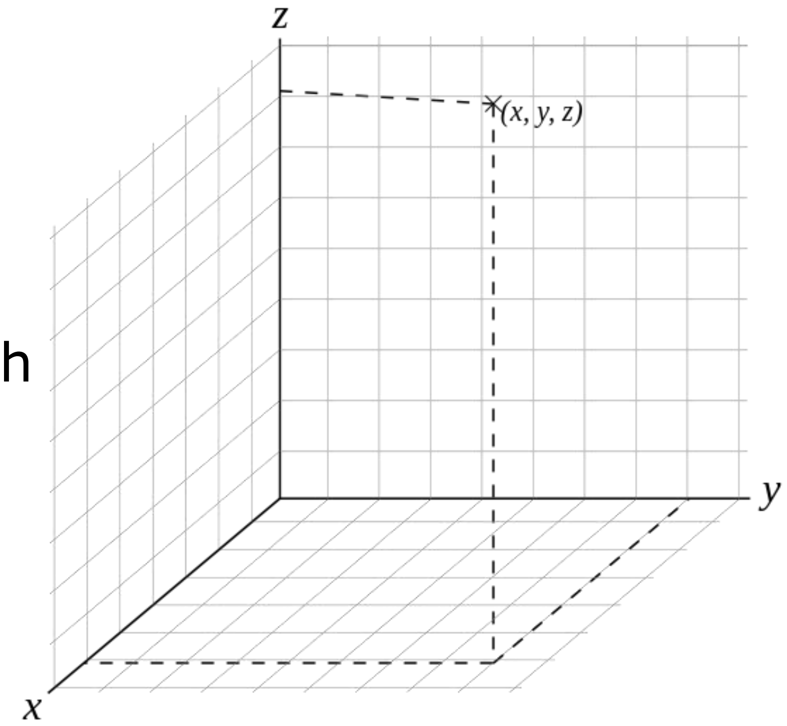
A RS serves to define in a rational way the position of the points on the physical surface of the Earth, assigning to each one its own coordinates, with a biunivocal relation between real numbers (coordinates) and the points themselves

First of all it is necessary to find a reference surface that represents well the Earth

Reference system

The concept of a reference system is intuitively known:

- Each drawing in AutoCAD has its own 2D or 3D reference system
- Cartesian reference system
- Origin of coordinates 0,0,0
- 3 orthogonal reference axes
- Oriented axes
- Origin of the axes corresponds with the origin of the system
- Measurement system



Reference system

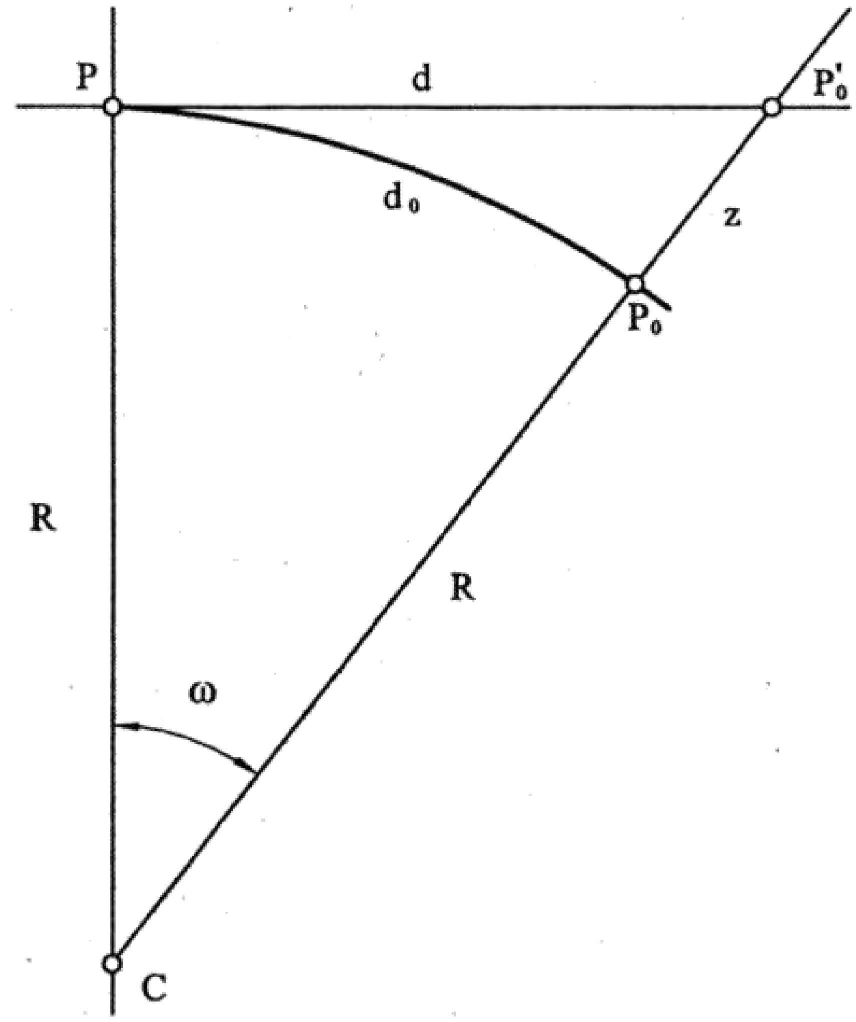
A Cartesian reference system is convenient for describing a limited portion of the earth's surface but becomes more complicated to use if larger portions of territory are to be described:

The terrestrial curvature can not be neglected => to take into consideration the actual form of the earth

The effect of terrestrial curvature can already be seen in the design of large structures, for example a stadium

The z coordinate in the Cartesian case is associated with the quote => in an extended system is no longer associated with the dimension idea

=> Need to define a different reference system !!!



Reference system

The reference system must have precise characteristics:

- Corresponding to the surface of the earth
- It must correspond to references of a physical nature
- It must be of simple mathematical expression

There is NOT a reference system that has at the same time the three characteristics

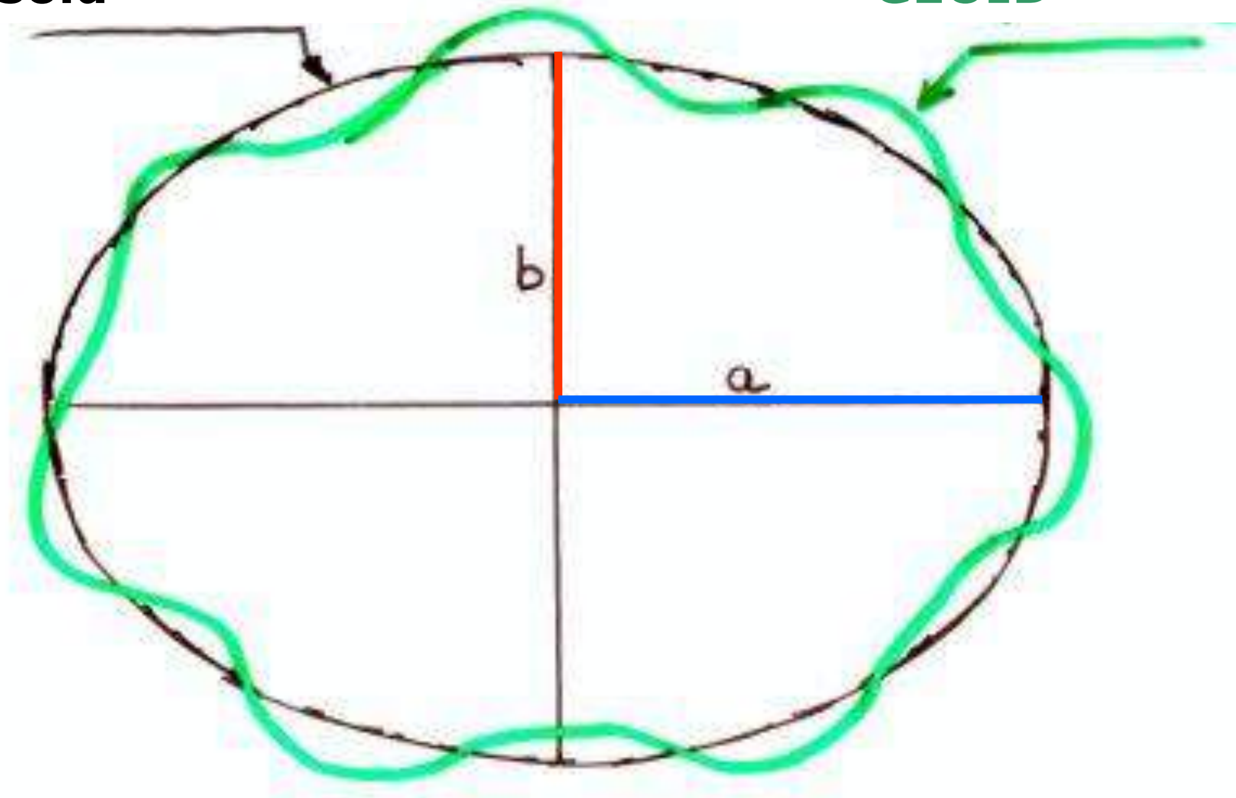
There is a historical division between altimetry and planimetry (physical reference - mathematical reference)

Today more and more integrated data

Il modello della Terra

Planimetry **Altimetry**

Ellipsoid **GEOID**



a: equatorial half-shaft

b: polar axle shaft

Models of the Earth

PLATE models:
Still used for flat representations on reduced distances (less than 10 km) for which it can be considered negligible the terrestrial curvature

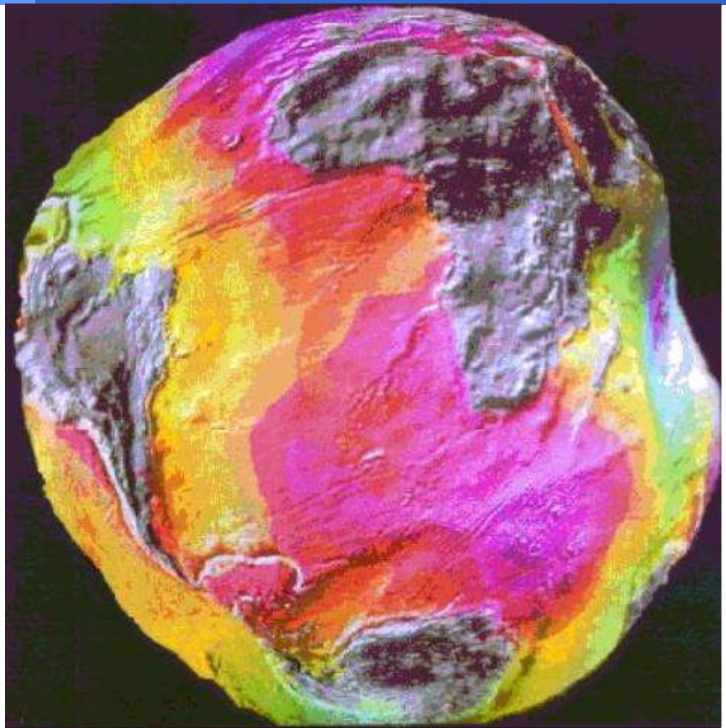
SPHERICAL models:
Used for local navigation and short distances when no particular precision is required. Problems arise when one wants to represent the whole planet. The slight crushing at the poles causes a difference of approx. 20 km between a sphere having a radius equal to the mean value of the planet and the radius measured at the poles. Therefore it is necessary to resort to ellipsoidal models.

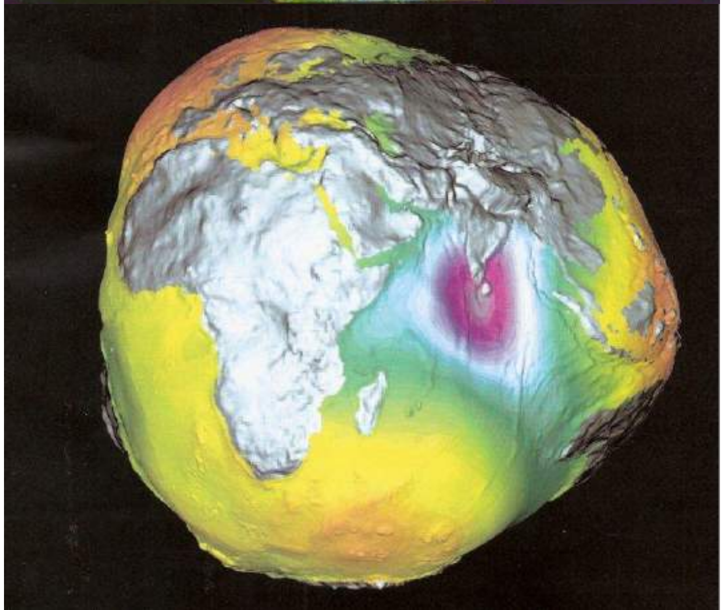
ELLIPSOIDAL models:
Used for accurate measurements over long distances. Recent satellite navigation systems (Loran-C and GPS) use models of this type.

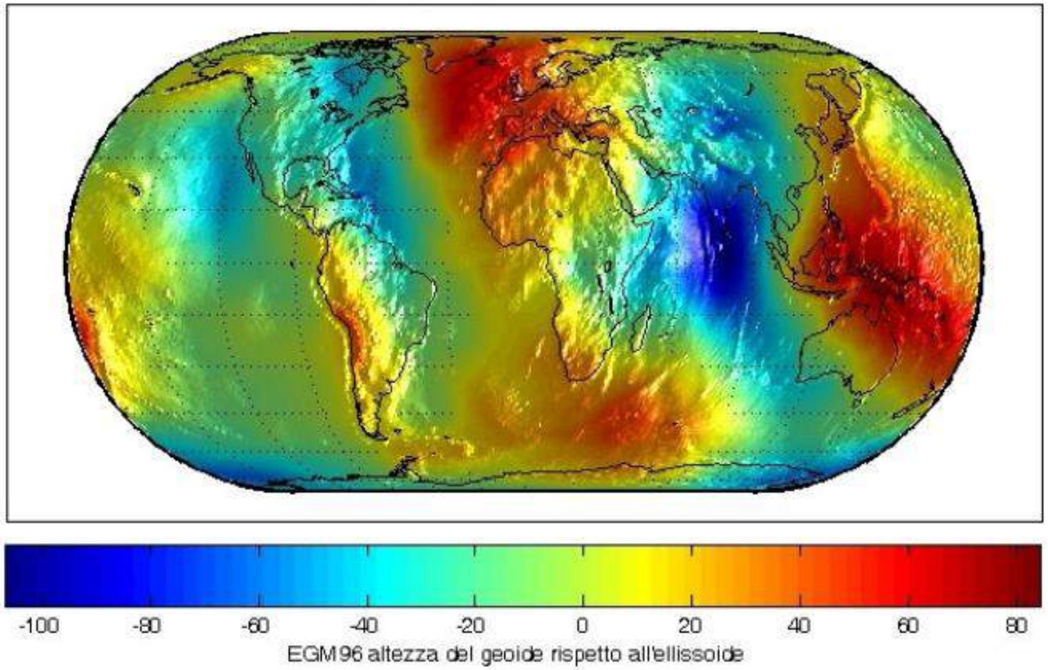
Geoid

- **Geoid** = reference area for altimetry
- As a matter of fact: water follows physical laws -> water flows from points of higher altitude towards points of lower altitude
- **Equivalent surface passing through the mean sea level**
- Point-to-point perpendicular surface to the plumb line that points to the center of mass of the planet
- The geoid would coincide with the surface of the seas, appropriately prolonged under the emerged lands, if the water of the seas had the same temperature, the same density and there were no perturbations due to currents, winds and tides. (G. Inghilleri)
- Surface of difficult mathematical determination => **geoid model**
- The orthometric height is defined on the mean sea level

Geoid





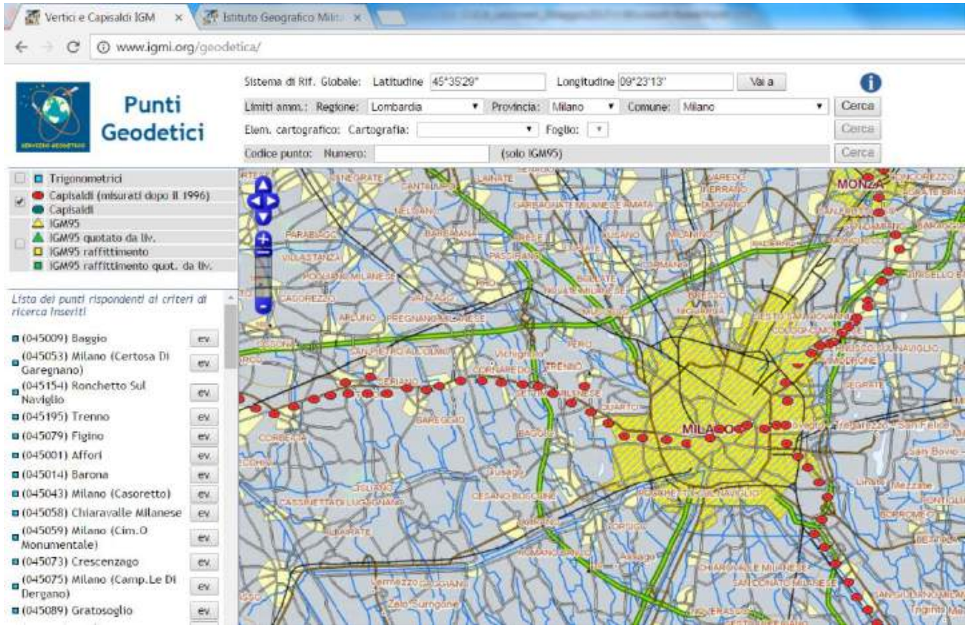


In the figure you can see a representation of the heights of the geoid EGM96 with respect to the ellipsoid GRS80.
The minimum is -106 m above the Indian ocean, while the maximum is about 85 meters.

The theoretical model becomes NETWORK through the materialization on the ground and a series of choices and conventions

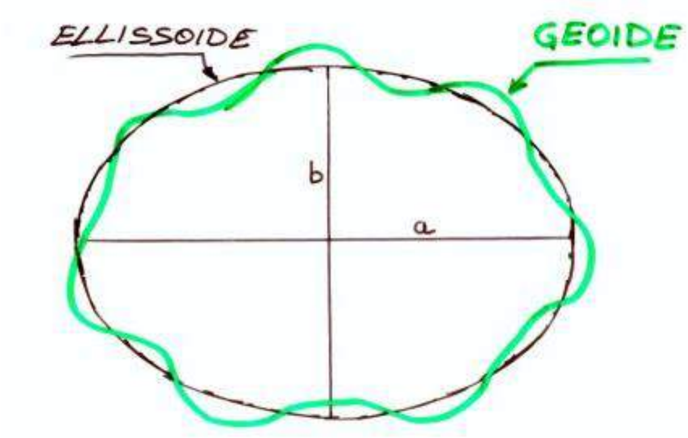
- tide gauges
- Altimetric leveling network
- European geopotential network

The share of benchmarks physically positioned on the territory is known



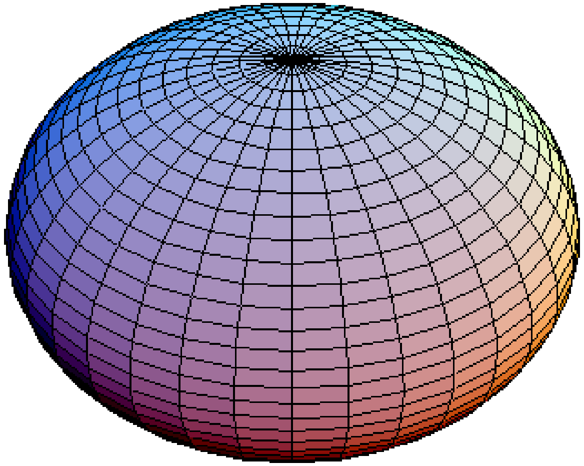
Ellipsoid

- The mathematical shape of the geoid is too complicated for cartographic applications (from a theoretical point of view the geoid is defined by an infinite number of parameters)



- It is shown that a good approximation of the geoid is an ellipsoid of rotation that, for cartographic applications, depends on only 2 parameters, mathematical form well corresponding to the shape of the earth

- Simple mathematical expression

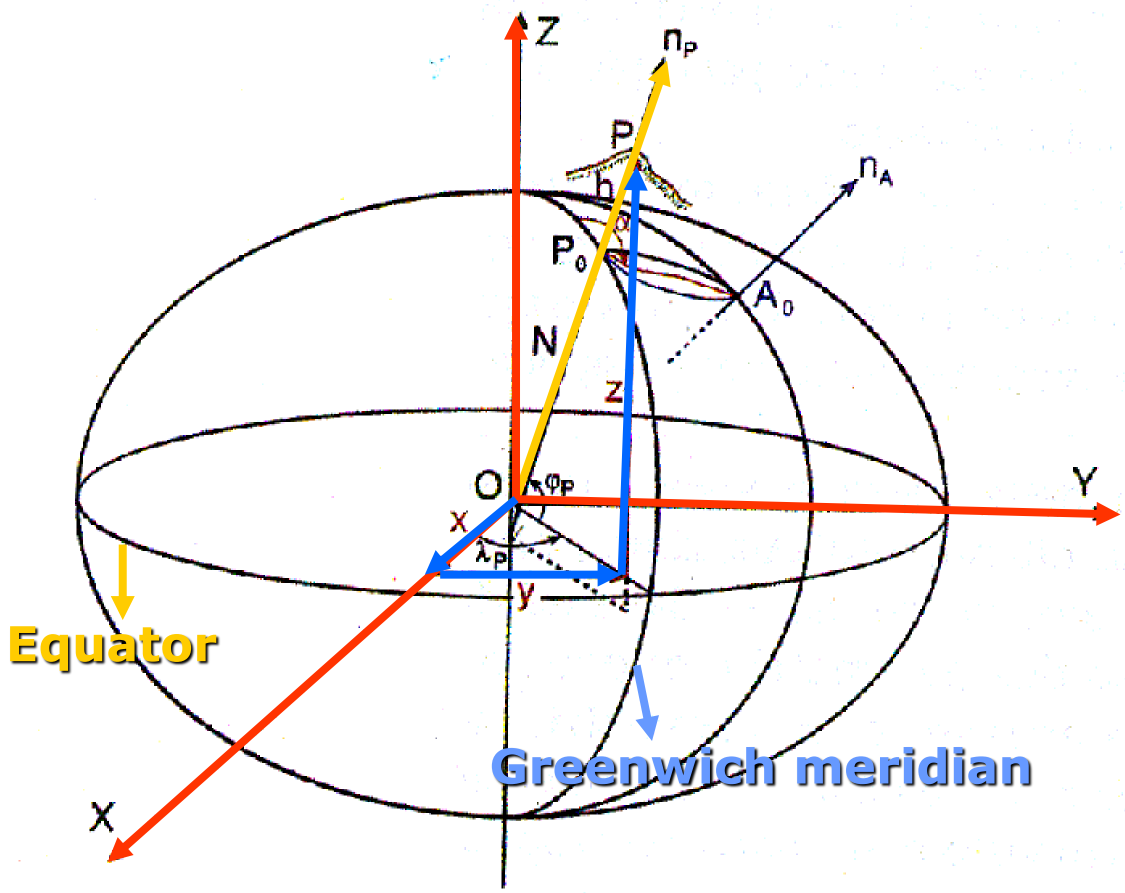


- Historical determination of the axes: equatorial (major axis - a) and polar (minor axis, rotation axis - b)

- Problems with the determination of the ellipsoid in history

- Italy - Ellipsoid by Bessel, Hayford and WGS84

Ellipsoid



$$\frac{x^2+y^2}{a^2}+\frac{z^2}{b^2}=1$$

Coord. P:

$$x=(N+h)\cos\varphi\cos\lambda$$
$$y=(N+h)\cos\varphi\,\mathrm{sen}\lambda$$
$$z=[N(1-e^2)+h]\,\mathrm{sen}\varphi$$

N = the first vertical of radius (large normal): the curve perpendicular to the meridian (intersection of ellipsoid and plane containing the normal n and the tangent to the parallel)

$$N=\frac{a}{\sqrt{1-e^2\sin^2\varphi}}$$

$$e^2=\frac{a^2-b^2}{a^2}$$

Ellipsoid parameters

$$\frac{X^2 + Y^2}{a^2} + \frac{Z^2}{b^2} = 1$$

$$\alpha = \frac{a - b}{b}$$

Bessel 1941	a = 6377397 m	α = 1/299.152813
Hayford 1909	a = 6378388 m	α = 1/297
WGS84	a= 6378137 m	α = 1/298.257223

Geocentric / geographical coordinates

Coordinates of Po

Projection of the P point on the ellipsoid

$$X = N \cdot \cos\varphi \cdot \cos\lambda$$

$$Y = N \cdot \cos\varphi \cdot \sin\lambda$$

$$Z = \left[N \cdot \left(1 - e^2\right)\right] \cdot \sin\varphi$$

Coordinates of P

The point on the earth's surface

$$X = \left(N + h\right) \cdot \cos\varphi \cdot \cos\lambda$$

$$Y = \left(N + h\right) \cdot \cos\varphi \cdot \sin\lambda$$

$$Z = \left[N \cdot \left(1 - e^2\right) + h\right] \cdot \sin\lambda$$

Parametri ellissoide

Le coordinate geodetiche di un punto T situato a Torino, riferite all'ellissoide WGS84, sono:

φ = *45° 03' 48",1186*

λ = *7° 39' 40",6046*

h = 310,764 m

I parametri dell'ellissoide WGS84 sono:

a = 6378137 m

a = *1/298,257223563 schiacciamento*

e^2 = *0,006694379990 eccentricità*

Determinare le coordinate cartesiane geocentriche:

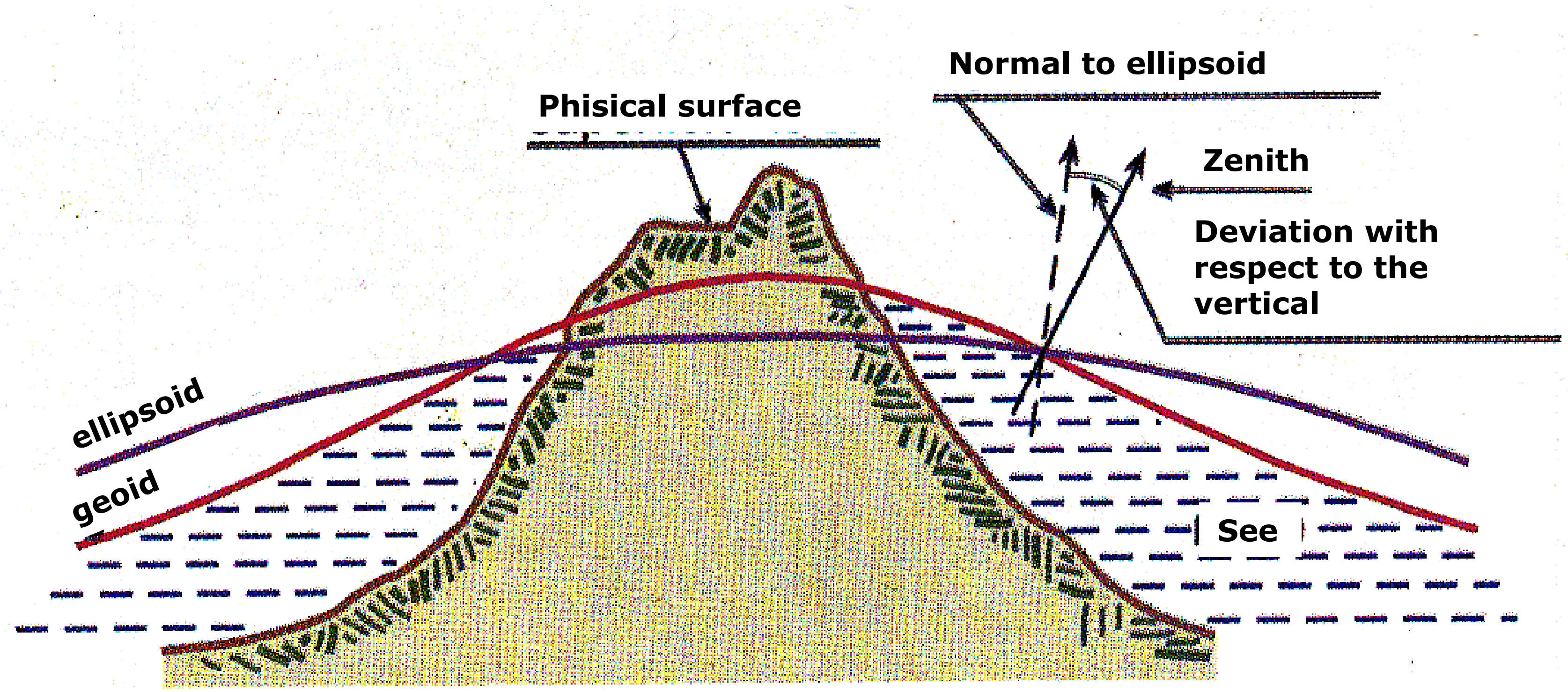
$$N_T = \frac{a_{WGS84}}{\sqrt{1 - e^2_{WGS84} \cdot sen^2 \varphi_T}} = 6388862{,}020m$$

$$X_T = (N_T + h_T) \cdot \cos\varphi_T \cdot \cos\lambda_T = 4472544{,}488m$$

$$Y_T = (N_T + h_T) \cdot \cos\varphi_T \cdot sen\lambda_T = 601634{,}185m$$

$$Z_T = \left(N_T \cdot \left(1 - e_T{}^2\right) + h_T\right) \cdot sen\varphi_T = 4492545{,}119m$$

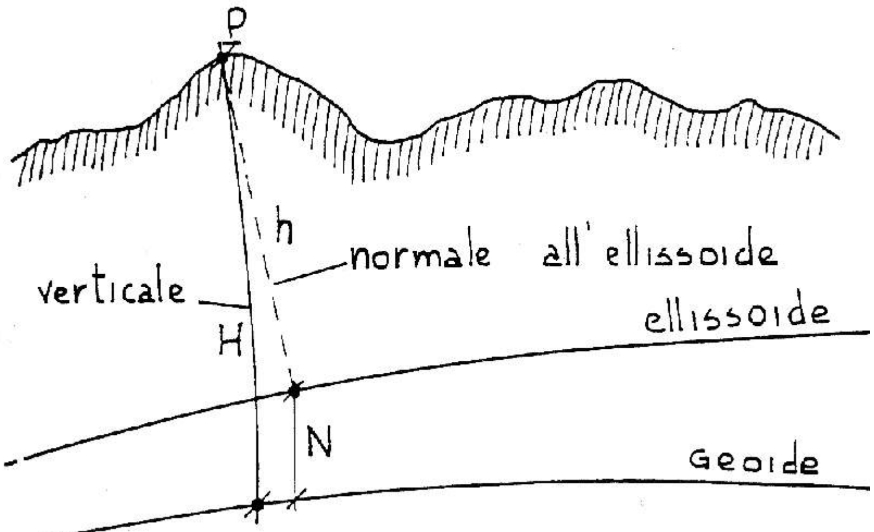
Orthometric vs ellipsoidal height



Orthometric vs ellipsoidal height

A point has two different dimensions:

- Ellipsoidal height (h): normal length to the ellipsoid or height of a point above the ellipsoid
- Orthometric height (H): length of the line of the force field of gravity or height of a point above the mean sea level or geoid



H= h + N

N: geoid undulation

Undulation

- In our areas N is worth about 30 - 40 m (attention, not cm !!!)

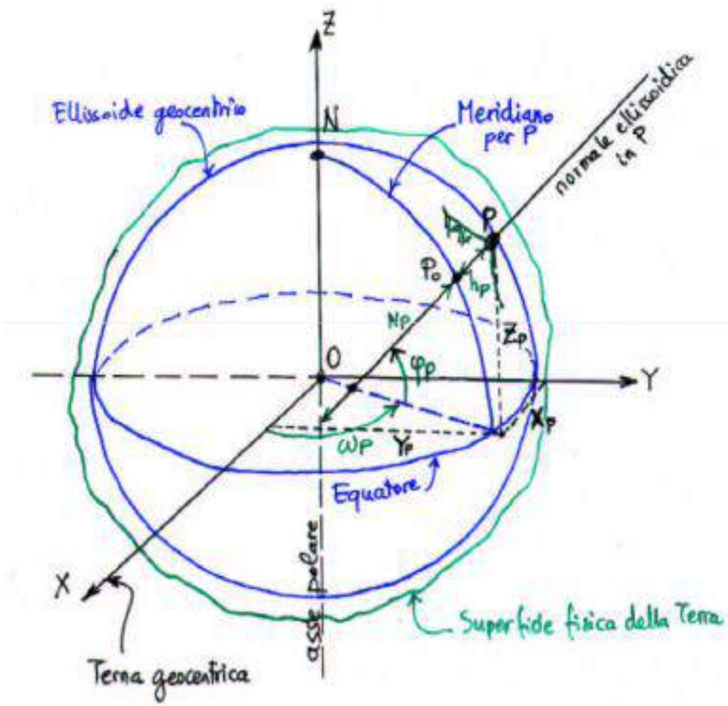
- In order to pass from H to h and vice versa, it is necessary to know the model of N (model of geoid) with accuracy of the order of 5-10 cm

Geographic ellipsoidal coordinates

We defined the geographic coordinates φ (latitude) and λ (longitude) by treating the geometry of the ellipsoid. The pair of values (φ and λ) defines the planimetric position of a point,
That is the position of the projection of the point
on the ellipsoid (in figure Po)
Altimetry is usually treated separately.

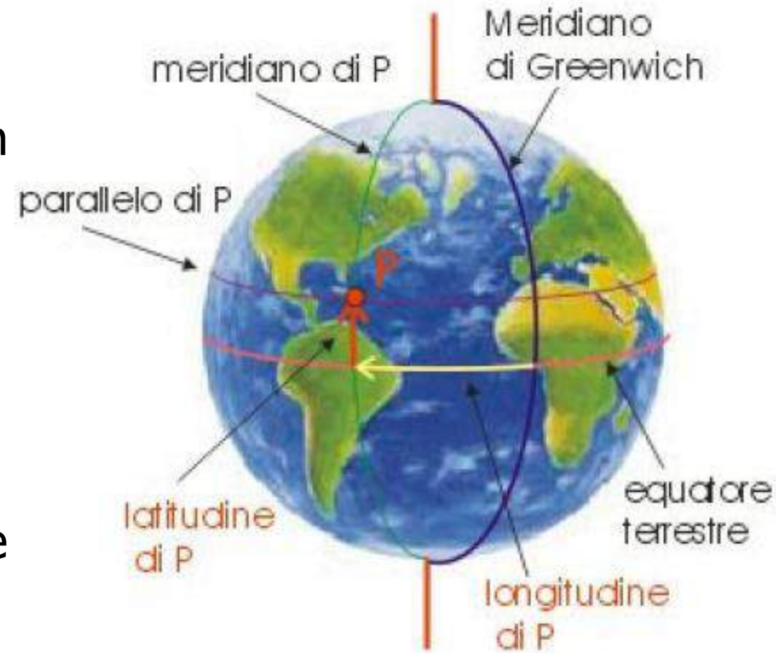
The triad (φ, λ, h) defines the three-dimensional position of a point.

Among all types of coordinates, geographic ones are generally used: they are used to provide
the results of the compensations of the networks (both classic and GPS trigonometric), to identify the
summits in monographs and catalogs, and for positioning and georeferencing problems in gender.

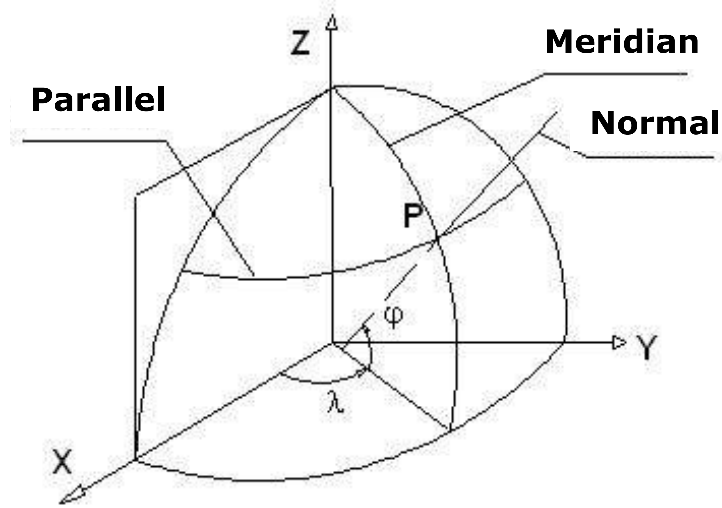


Geographic ellipsoidal coordinates

In the terrestrial coordinate system, the equator is chosen as the fundamental plane, while the fundamental direction is the rotation axis of the Earth. It is assumed that the earth's surface is, in first approximation, spherical in shape.



Any plane that contains the earth's axis (meridian plane), determines on the earth's surface a maximum circle passing through the poles called the meridian circle.
By geographic meridian one means one half-circonference between the two poles and each meridian has its own antemeridian that completes the meridian circle, on the opposite side. The meridians are all the same.

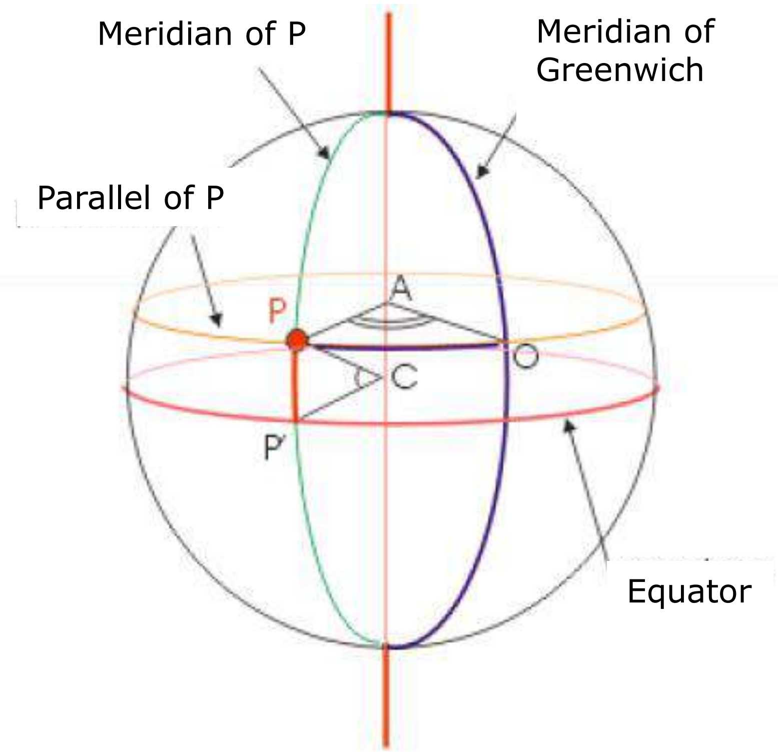


Parallels, on the other hand, are the circles formed by the intersection of any plane parallel to the equator with the earth's surface. The parallels are the smaller the greater their distance from the equator.

Geographic ellipsoidal coordinates

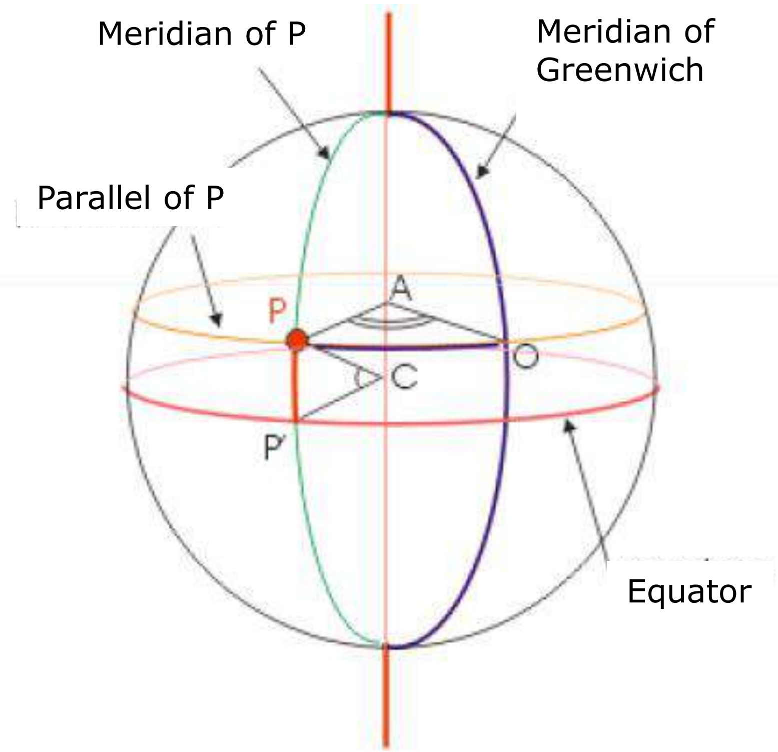
Parallels and meridians form a network on the surface (geographic crosslink), which allows us to identify the absolute position of a point. To do this, just indicate the parallel and the meridian that pass through this point (parallel to the place and the meridian of the place). In order to indicate a precise parallel or meridian, the geographical coordinates are defined.

The geographical longitude (λ) is the angular distance of a point from the fundamental meridian (or Greenwich), measured on the parallel arc passing through that point: it is the angle between the meridian plane of the point and the plane of the fundamental meridian. In the drawing on the side, this is the angle PAO where A is a point on the terrestrial axis belonging to the parallel plane of P.



Geographic ellipsoidal coordinates

The geographical latitude (φ) is the angular distance of a point from the equator measured along the meridian passing through that point. It corresponds to the angle between the vertical of the place and the plane of the equator. In the drawing this is the PCP angle 'where C is the center of the Earth. It ranges from + 90 ° (north pole) to - 90 ° (south pole). The points along the equator they have latitude 0 °.



Both longitude and geographical latitude are expressed in degrees and fractions of degree.

The parallels can be considered sets of points on the earth's surface that have equal latitude and the meridians sets of points with equal longitude. Meridians and parallels are infinite, but it is often used to take into consideration those that they are one degree apart from one another. They are called degree meridians and degree parallels.

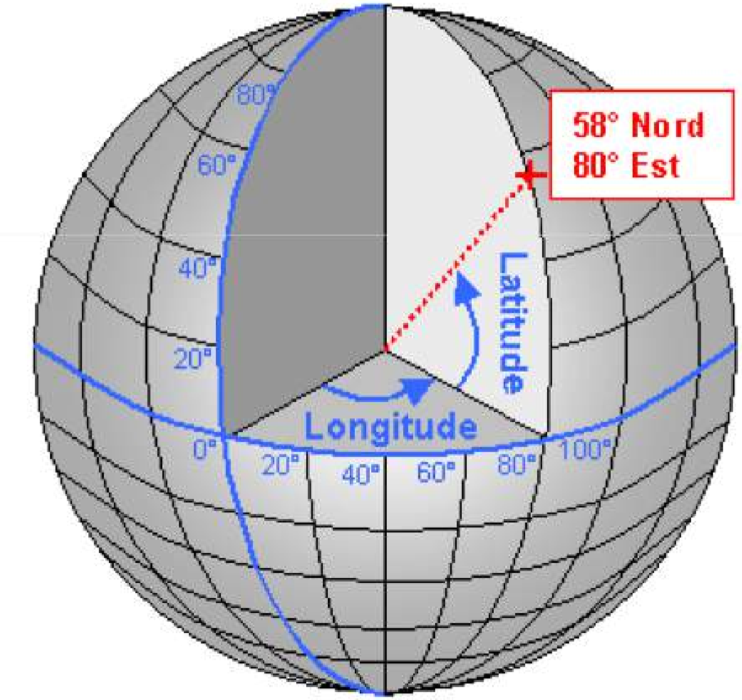
There are 360 degree meridians and 178 degree parallels (excluding the two parallel to the poles, which are reduced to one point).

Geographic ellipsoidal coordinates

Latitude and Longitude are traditionally measured in degrees, minutes and seconds.

The latitude varies between -90 ° (South Pole), 0 ° (parallel coinciding with the equator) and 90 °(North Pole).

The longitude has the coincident 0 ° axis with the meridian that starts from the North Pole, it passes from Greenwich, England (in 1884 it is chosen as meridian origin), and ends at the South Pole.
It is measured positively, up to 180 ° starting from Greenwich to the east e negatively up to -180 ° traveling from Greenwich to the west.

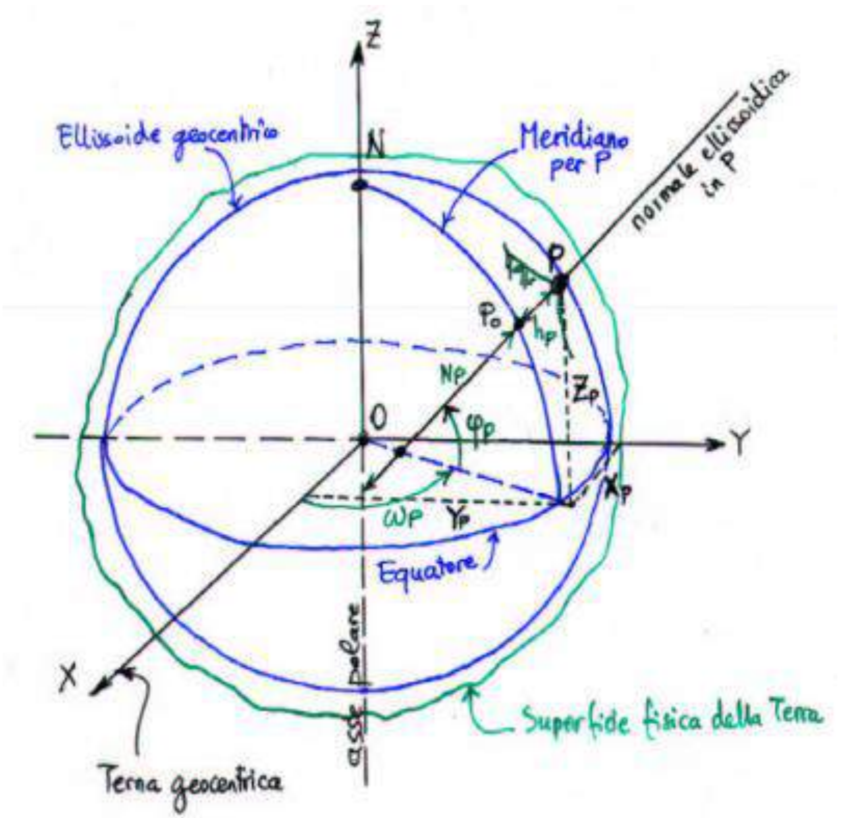


Geographic geocentric coordinates

Geocentric coordinates (X, Y, Z) are the Cartesian coordinates of a point with respect to the three axles geocentric OXYZ.

The set of values (X, Y, Z) defines the three-dimensional position of a point in a completely equivalent to triplet (φ, λ, h) referred to the ellipsoid geocentric, having the axes along the directions X, Y, Z.

Geocentric coordinates identify the three-dimensional position of a point without equivocal.



Geographic coordinates ???

- Working in ellipsoidal or geographic coordinates is convenient because there is the uniqueness of a point on the earth

- It's inconvenient because we are used to thinking in flat systems for planning, planning, ...

- The idea of inventing flat (local) systems is born

- The planimetric position of a point can also be expressed by means of its plane coordinates in any cartographic representation. Since the cartographic representation establishes a bi-univocal correspondence between the ellipsoid and the plane of the map, the cartographic plan coordinates are practically completely equivalent to the geographical coordinates (φ and λ), to which they can be traced back to the map equations and the inverse ones.

- Because of their practicality of use, the cartographic coordinates are the most used to define the planimetric positions in the surveys, both in the photogrammetry and in the ground survey with traditional topographic or GPS techniques.

- Introduction of "flat cartography" deformations must be introduced

Cartography

- Need to move to a plan support - cartographic plan
- Cartesian coordinates are based on a system of two orthogonal axes (x, y). The location of a point given by the intersection of the two values. The reference axes are the equator and a fundamental meridian which, usually, is not the one used for geographic coordinates.
- We must introduce deformations (angular, distances, areas)
- Projection and representation methods
- The geographic coordinates (λ, φ) and the planimetric coordinates (x, y) are measured on the surface of the reference ellipsoid.
- Need to define analytical formulas to pass from (φ, λ) to (x, y) or (East, North)
- Among the infinite solutions I choose the one that keeps deformations as small as possible

$$X = f(\varphi, \lambda)$$

$$Y = g(\varphi, \lambda)$$

Cartography

$$m = \frac{ds^I}{ds}$$

Linear deformation module

Equidistant map **m=1**

$$\mu = \frac{d\sigma^I}{d\sigma}$$

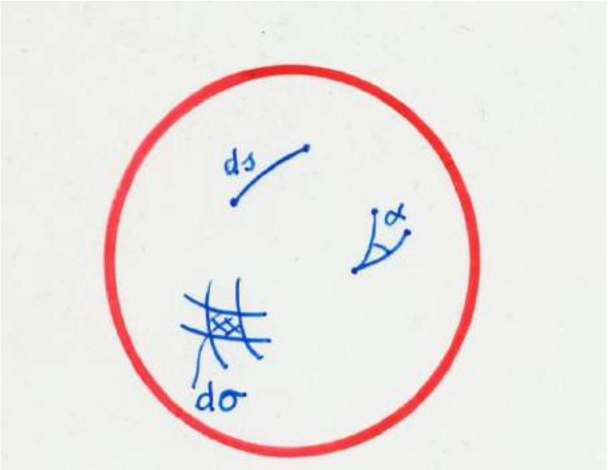
Surface deformation module

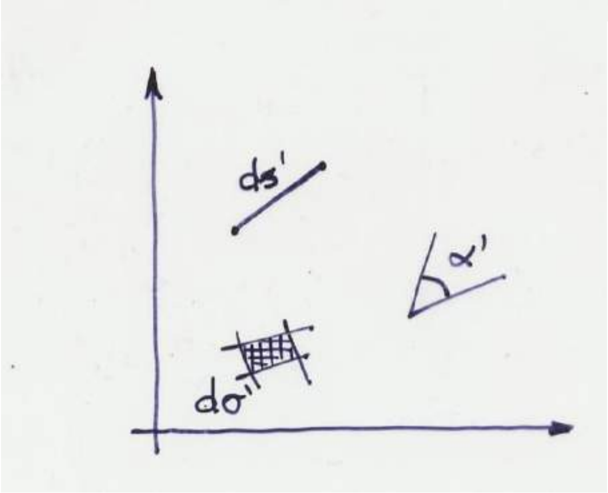
Equivalent map **μ=1**

$$\delta = \beta^I - \beta$$

Angular deformation module

Conformal map **δ=0**



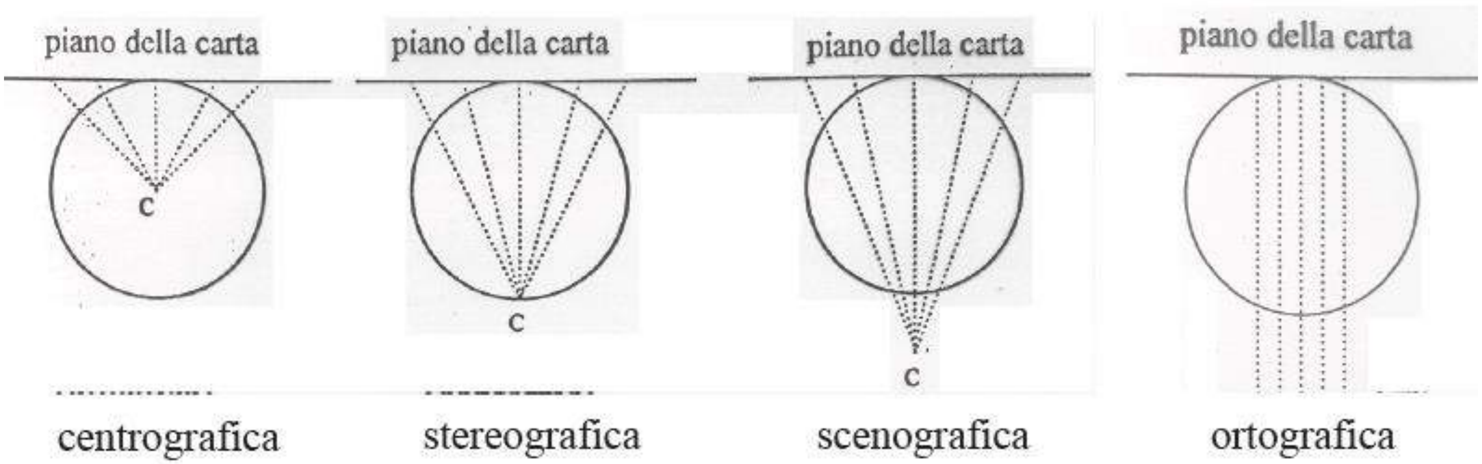


The scale factor is to be used separately. In fact, it is possible to reduce the ellipsoid and then make the projection or to make the projection and then make the map smaller.

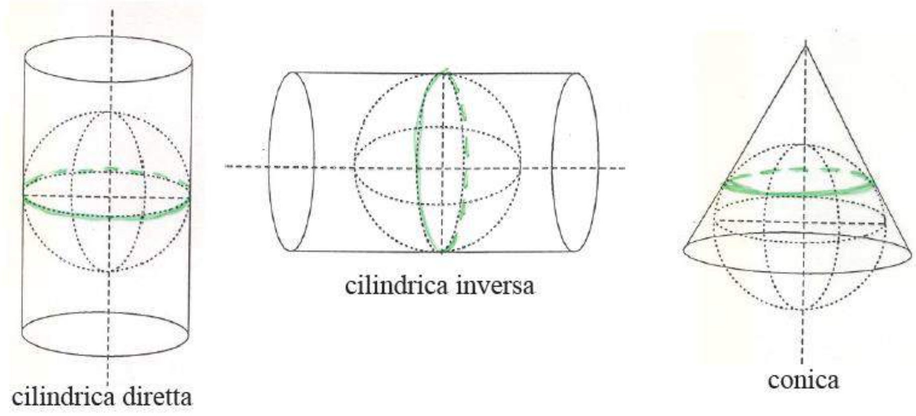
Cartographic representations

Once the reference surface is fixed, the map can be created

Maps can be created by projection, i.e. projecting the points directly from the surface to the plane

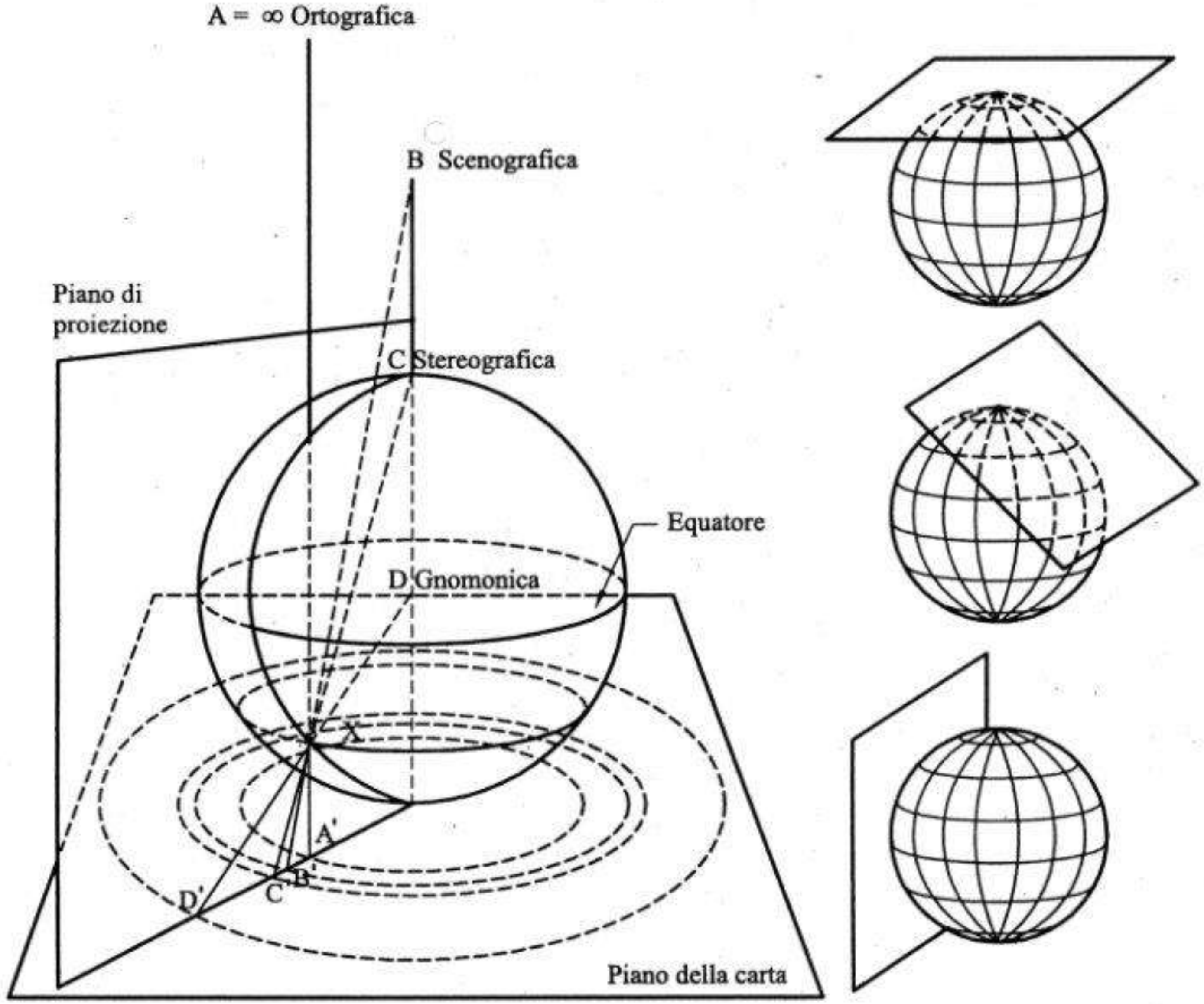


Maps are also created for unfoldinf, i.e. using a surface that will then be "spread" on the plane



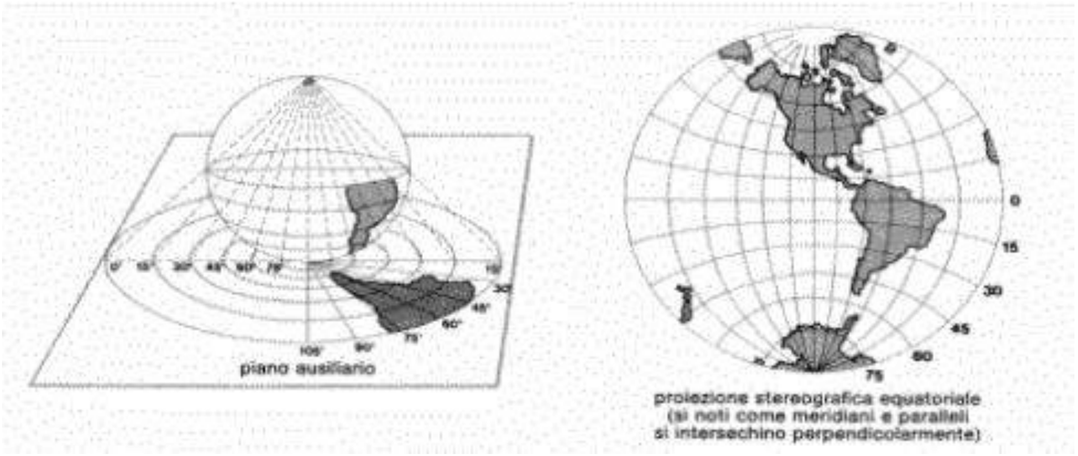
Direct projections

- Project directly on the plane from a point
- It is always possible to write the corresponding analytical reports
- I can choose the projection point and the position of the projection plane


A = ∞ Ortografica
B Scenografica
Piano di proiezione
C Stereografica
Equatore
D Gnomonica
X
A'
B'
C'
D'
Piano della carta

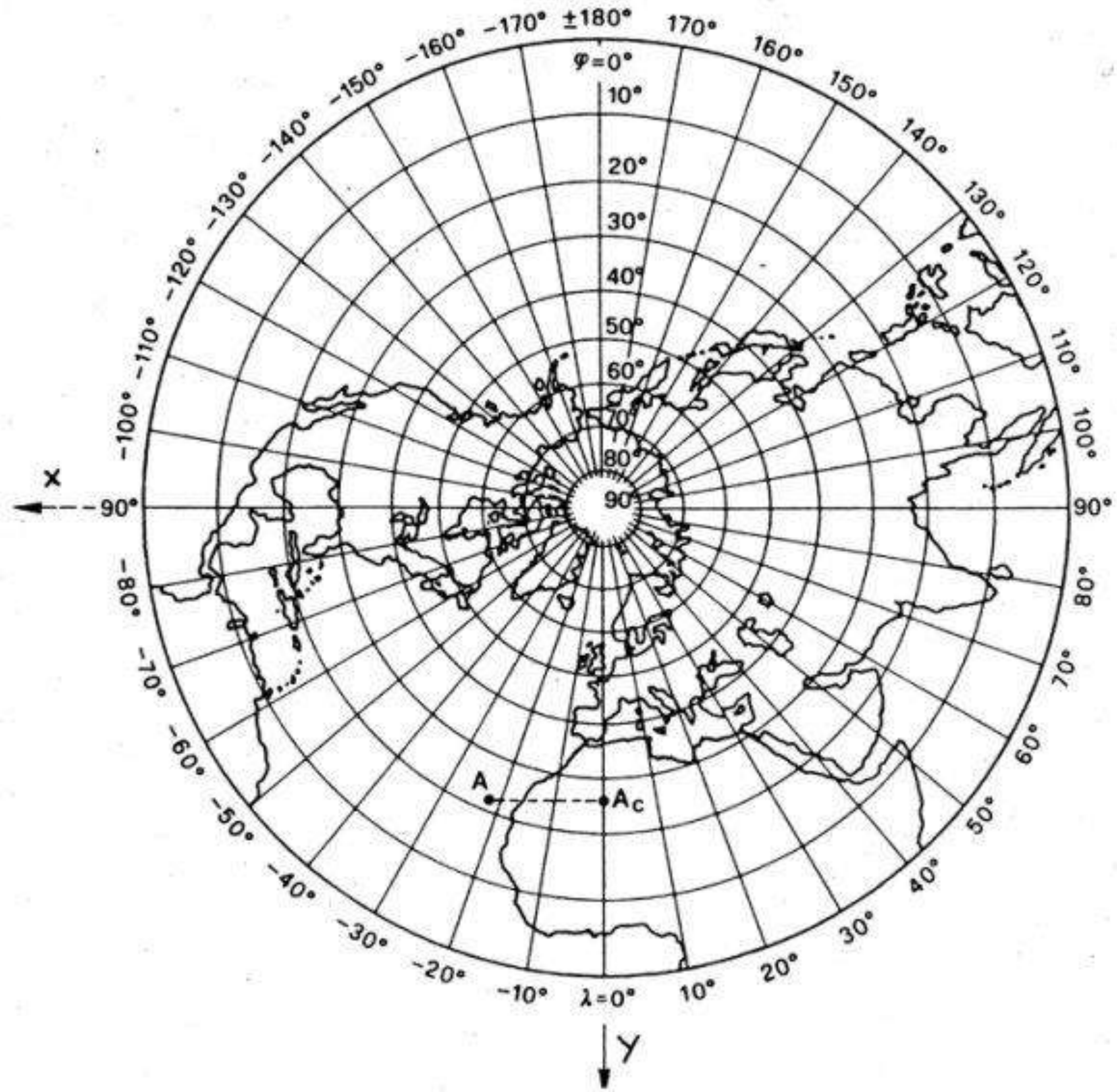
UPS

- Azimuth projection: the plane of the map, or picture, is tangent to the sphere at a point called the projection center.
- Stereographic point of view, the projection takes place from the opposite pole.



- A direct projection is chosen as a universal projection relative to the polar ice maps
- UPS - Universal Polar Stereographic
- Valid above +82 and below -80 latitude
- Unused territories (the linear deformation coefficient is high)

UPS



±180°
φ=0°
10°
20°
30°
40°
50°
60°
70°
80°
90°
-170°
-160°
-150°
-140°
-130°
-120°
-110°
-100°
-90°
-80°
-70°
-60°
-50°
-40°
-30°
-20°
-10°
λ=0°
10°
20°
30°
40°
50°
60°
70°
80°
90°
100°
110°
120°
130°
140°
150°
160°
170°
X
Y
A
A_C

UPS

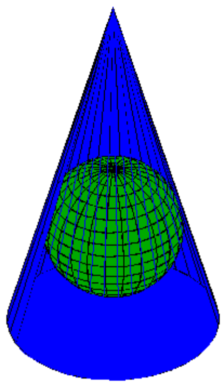
- m = 1 at the point of contact
- The map is conforal - it supports the orthogonality between meridians and parallels
- Parallels are concentric circles with irregular intervals
- The meridians are straight lines outgoing from the pole

Conical projections

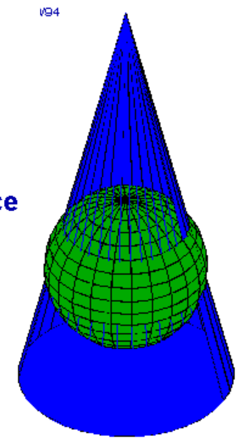
The lattice is projected into the inner part of a tangent cone or secant the globe along a minor circle, generally at mid latitudes.

The parallels are projected as concentric circle arcs.

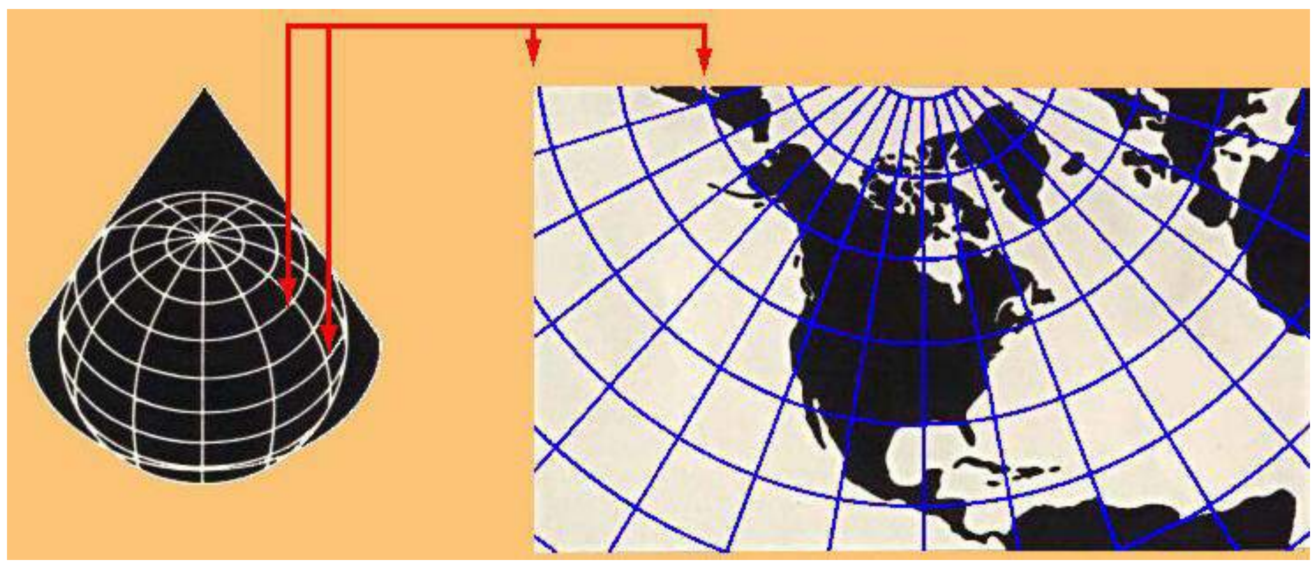
The meridians are instead projected as straight lines that radiate from the central area and spaced with regular angles.



Conical Projection Surface



Secant Conic Projection



Conical projections

Among the most common are the secular conical projections, produced starting from two standard parallels.

The scale of the maps obtained from these projections increase their distortion with increasing distance from the standard parallels.

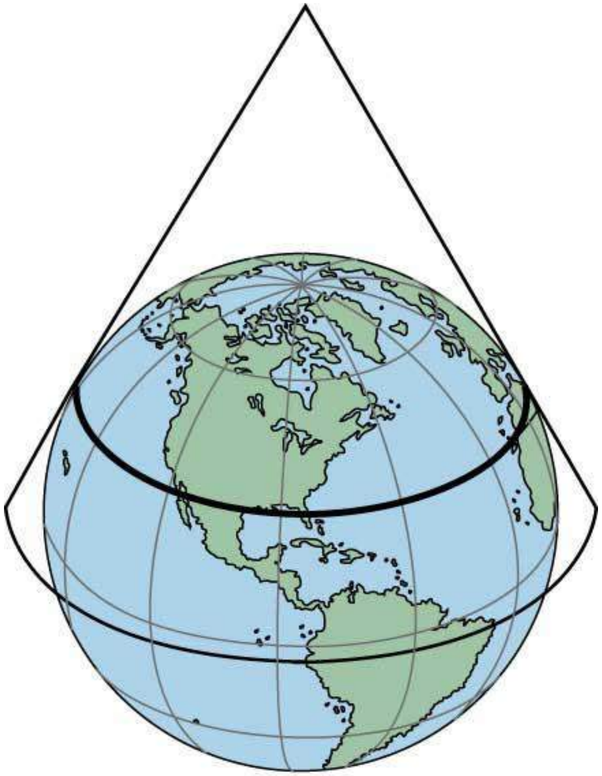
Conical projections are mainly used to represent regions at mid-latitudes and especially those that have an East-West distribution

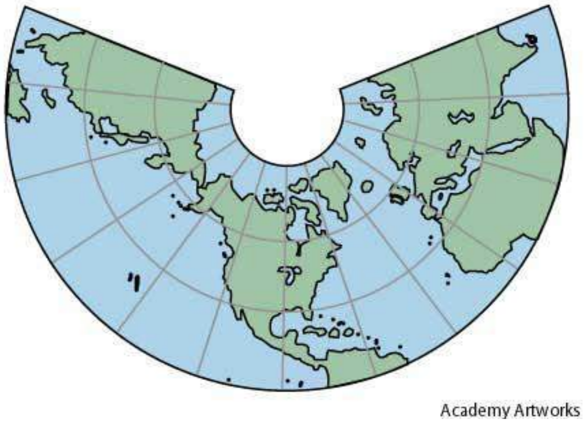
Variation of the way in which the standard parallels and the other constants are chosen

Variation in the spacing of the parallels to obtain arbitrary distortion compromises

Adaptation of conical projections to both the sphere and the ellipsoid

Development of pseudoconic projections, such as the projection of Bonne or others, which are not true conics

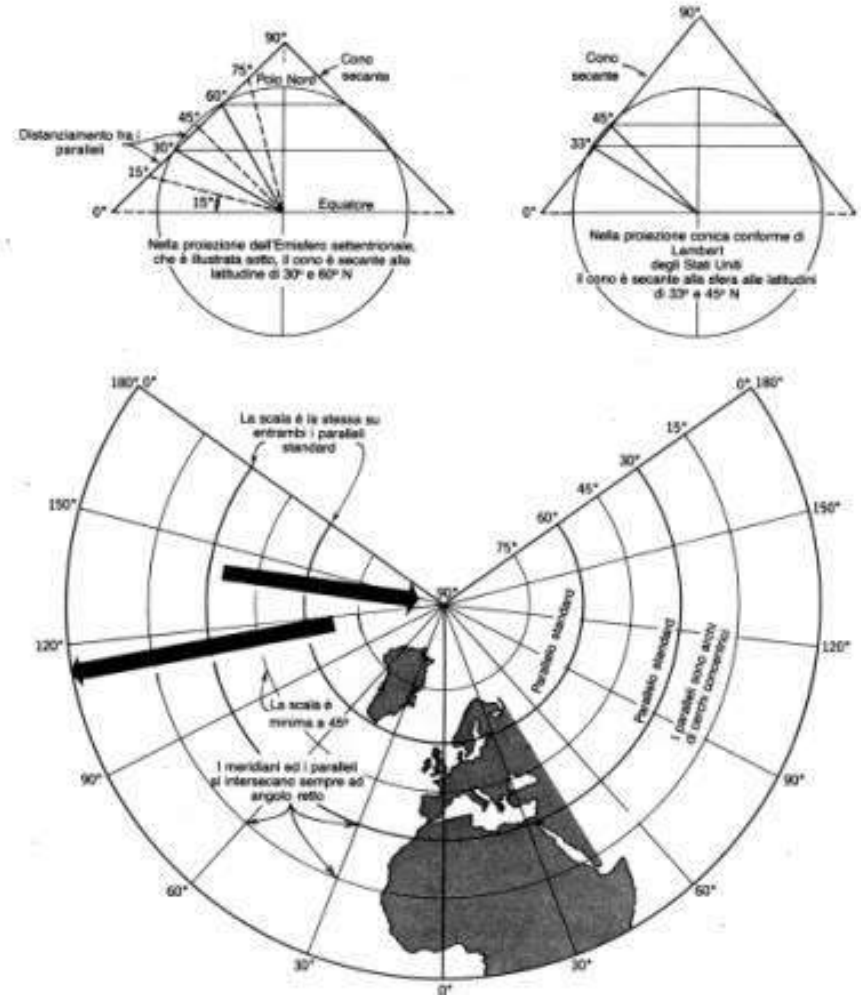


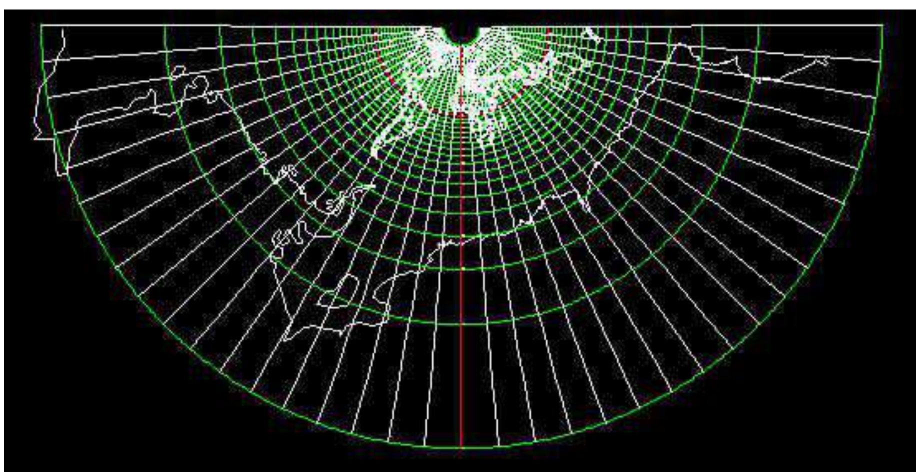


Conical projections

Lambert conformal projection.

It is a perspective (conical) projection with two standard parallels; in this way an improvement of the scale characteristics is obtained. This is a conformation representation used above all in air navigation.





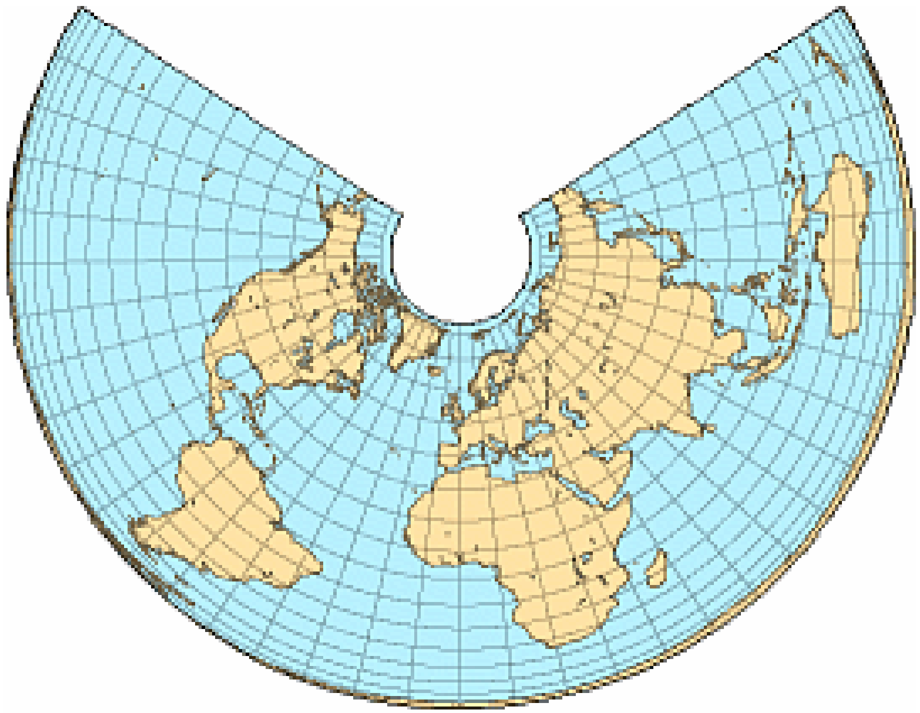
Proiezioni coniche

Conica, Alberts

Heinrich C. Albers (1805)

It maintains the relationships between the areas unchanged.
Albers (1773-1833), born and lived in Luneburg (Germany) proposes formulas for the projection of a sphere using two standard parallels.

Distortion is minimal along the central parallel and nothing on the standard parallels.
If the standard parallels are located in the two hemispheres and are equidistant from the equator, the projection becomes cylindrical equalarea.



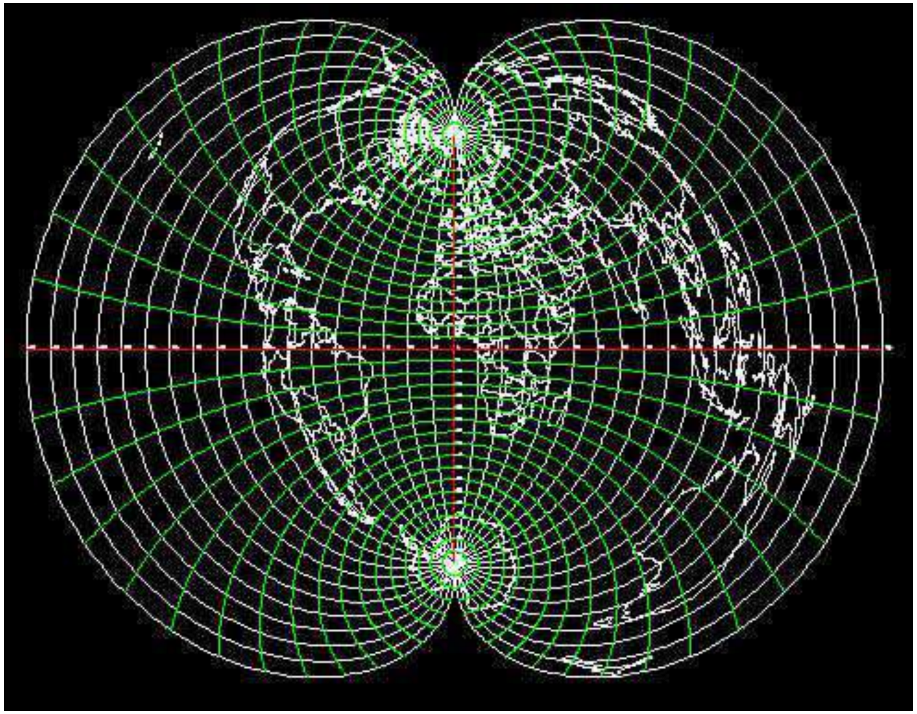
Esempio
Standard parallels: 60°N e 30°N;
Central meridian 0°

Proiezioni coniche

-**Policonica** (proiezione con archi circolari per paralleli di latitudine che non sono concentrici)

-Né conforme, né equalarea

Le proiezioni policoniche sono applicate come proiezioni specifiche nel 1853 da Edward Bissel dell'U.S. Coastal survey a partire da una proposta di Ferdinand Rudolph Hessler (1770-1843).
Viene comunemente ma non esclusivamente usata per le carte costiere degli USA.
Quando nasce l'U.S. Geological Survey e inizia la pubblicazione delle mappe geologiche la proiezione policonica era l'unica a disposizione delle agenzie topografiche e continua ad esserlo fino alla prima metà del 20° secolo.

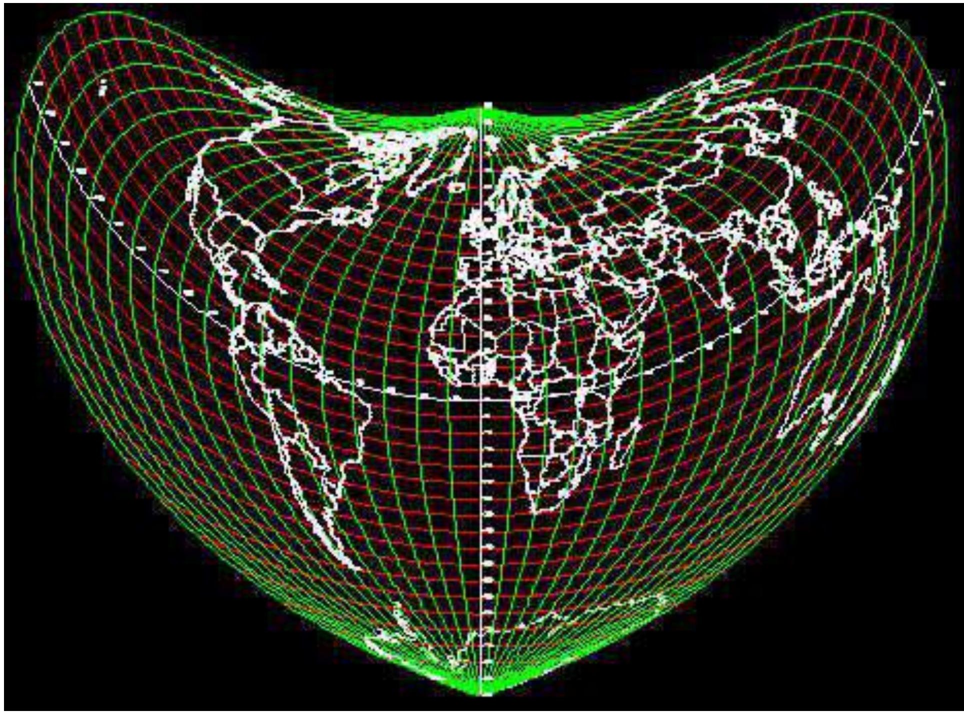


La proiezione policonica di Hessler è applicabile sia alle sfere sia agli ellissoidi. Non è conforme né equalarea,
È libera da distorsioni lungo il meridiano centrale.
Viene solitamente utilizzata nelle regioni con estensione N-S

Proiezioni coniche

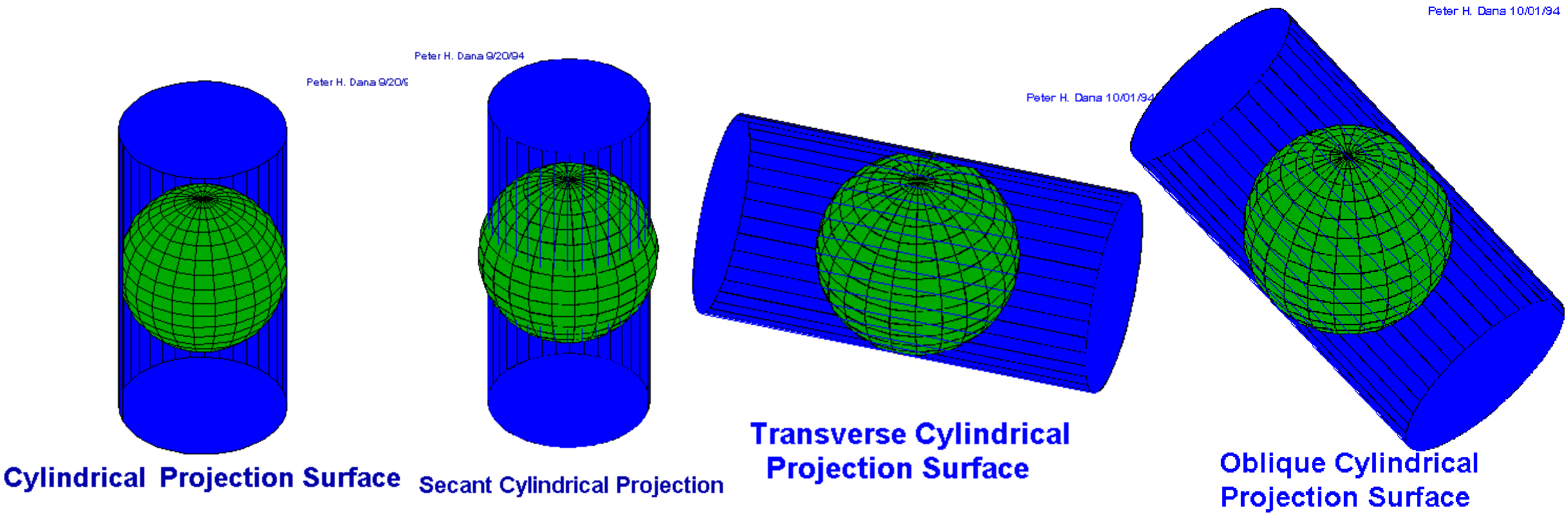
The Bonne projection plays an important role in the 19th century for mapping large regions using the ellipsoid formula. Its use began in France when Rigobert Bonne (1802) developed it. With this projection some European countries are represented as Austria-Hungary (1: 750.000), Belgium (1: 20.000), Denmark (1: 20.000), Italy (1: 500.000), Holland (1: 25.000), Russia (1: 126,000), Spain (1: 200,000), Switzerland (1: 25,000 and 1: 50,000), Scotland and Ireland (1: 63,360 and minors) and France (1: 80,000 and 1: 200,000).

It has also been used by the U.S. Coastal Survey for the Delaware Bay and for the representation of Virginia at the scale of 5 miles per inch (1: 316.800).



Cylindrical projections

They can be imagined as a simple N-S oriented cylinder that envelopes our Planet tangentially to the Equator. If the latitude and longitude lines were projected on the inside of the cylinder, a network of straight lines would cross each other. The meridians of longitude, parallel, would be regularly spaced, but the parallels of latitude would also remain parallel but would not present a regular spacing.



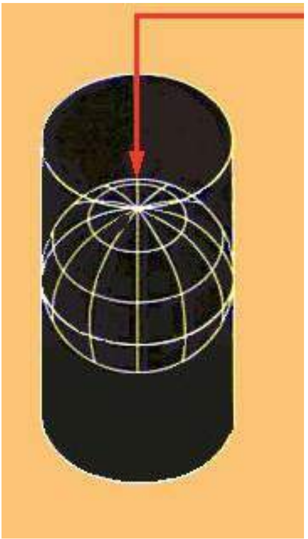
Cylindrical projections

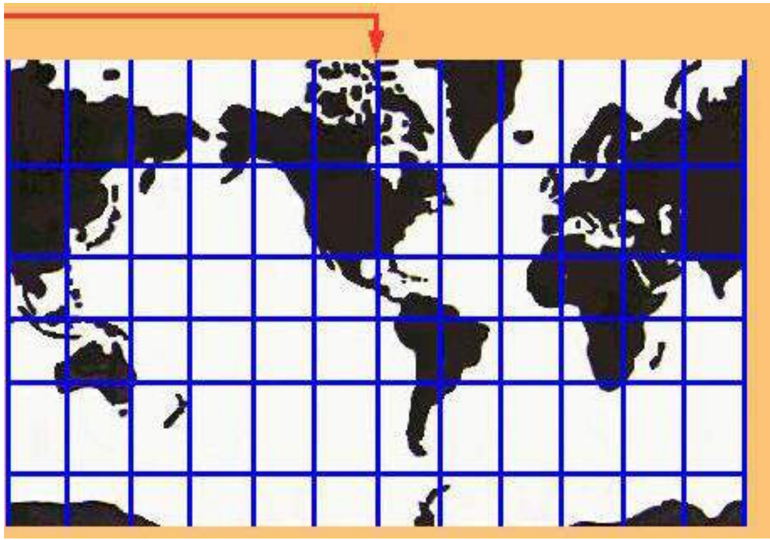
Mercator projection

Features:

-Conformal

-The meridians remain equidistant, while the parallels, moving from the Equator to the Poles, move away from each other in proportion to how much the distance of the meridians is increased on map compared to reality. This projection makes the map conform, but the surfaces become more and more deformed as they approach the Poles (for example, Greenland appears larger than South America).





The Mercator map has some flaws:

- can not be used at high latitudes because it deforms the area to be exaggerated
- can not represent polar areas. Generally used to map equatorial regions, oceans and the world in the 19th century.

Cylindrical projections

Miller cylindrical projection

Osborn Maitland Miller, 1942.

Miller's cylindrical projection is a modified Mercator projection. The projection formula is that of Mercator.

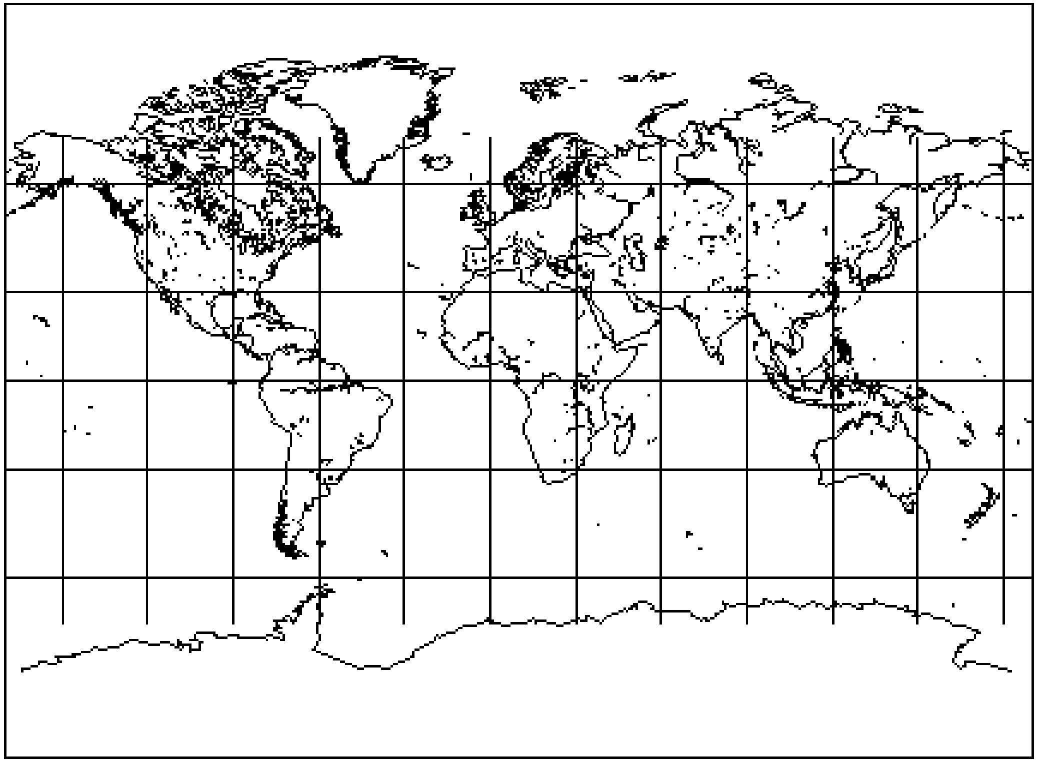
The parallels are spaced differently, the distance between them increases as you approach the poles, however this distance is always smaller than that of the direct projection of Mercator. This means that the poles can be represented.

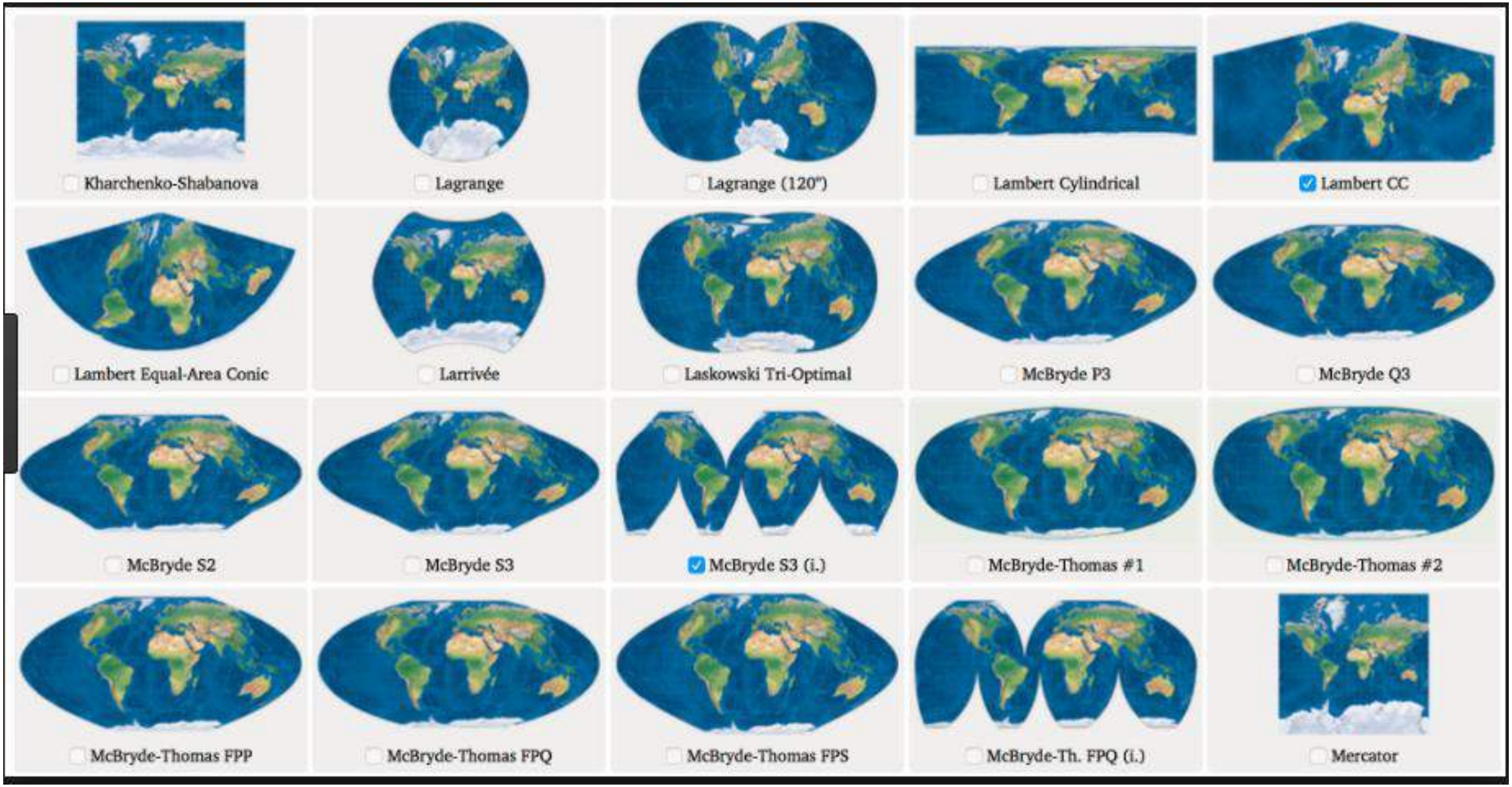
Miller's projection is neither equivalent or equidistant, neither compliant nor prospective.

The scale is only true long the equator.

The distortions become unacceptable at the poles.

This projection avoids scale exaggerations of the Mercator projection.





Sorce: **Compare Map Projections**
http://map-projections.net/imglist.php

Cylindrical projections

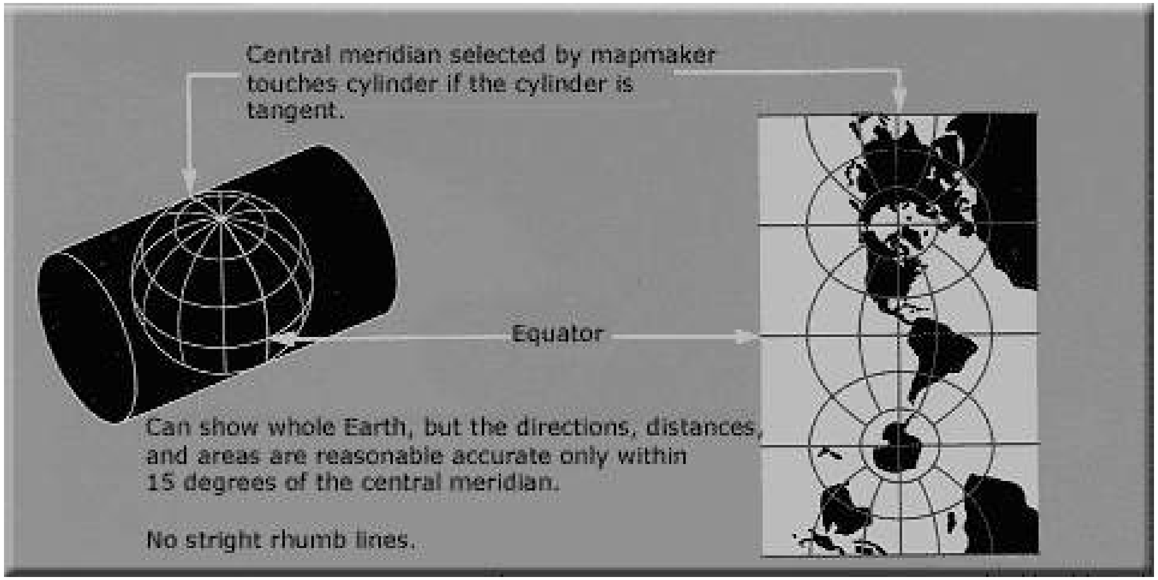
Mercator transverse projection

- Cylindrical cartographic projection (Mercator projection rotated 90 °). The projection cylinder is perpendicular to the earth rotation axis. The parallels are therefore equally spaced.

-Applied mainly to spherical representations

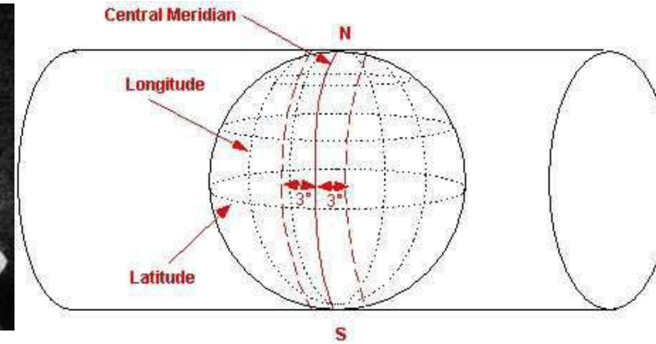
-Conforal

Invented in 1772 by Lambert, modified for the ellipse by Gauss in 1822 and by Kruger in 1912. Used for regions with N-S expansion. Used for topographic maps.



Cylindrical projections

- **Gauss Map** (1820): cylindrical transverse projection, the axis of the cylinder is orthogonal to the rotation axis of the Earth. Gauss map is build **analytically** (mathematically) by imposing these conditions:
  - The map has to be conformal
  - The central meridian (used as the origin for the longitude) has to became the abscissa axis (N)
  - The equator has became the East axis (E)
  - A portion of arch length *l* on the meridian has to became a segment of equal length on the N axis (representation is equidistant on the central meridian)
  - Considering a point P an angle α formed by two directions starting from P on the ellipsoid has to be maintained on the map
  - Linear deformation modulus, even if varying pointwise, for each point P ha to be equal in all directions



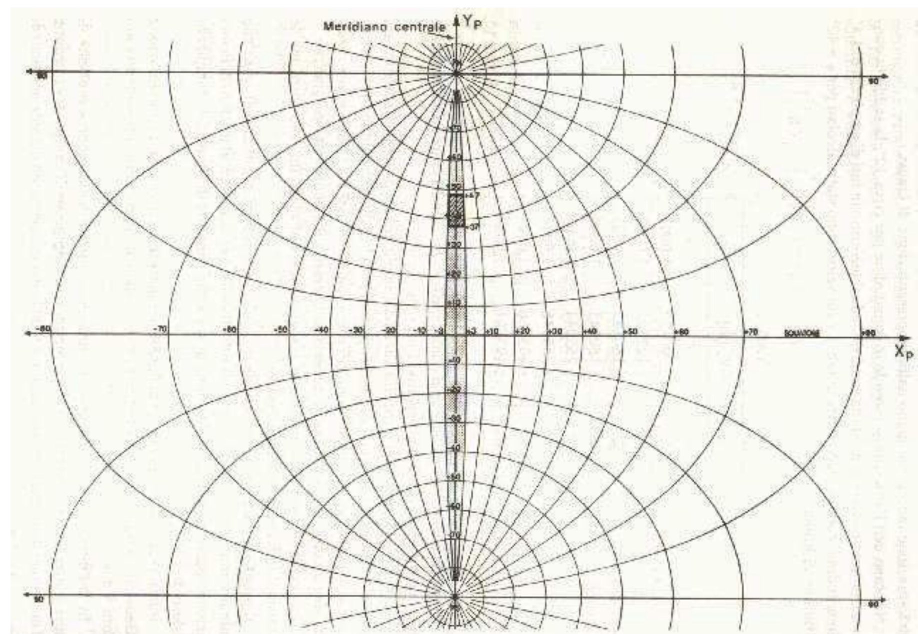
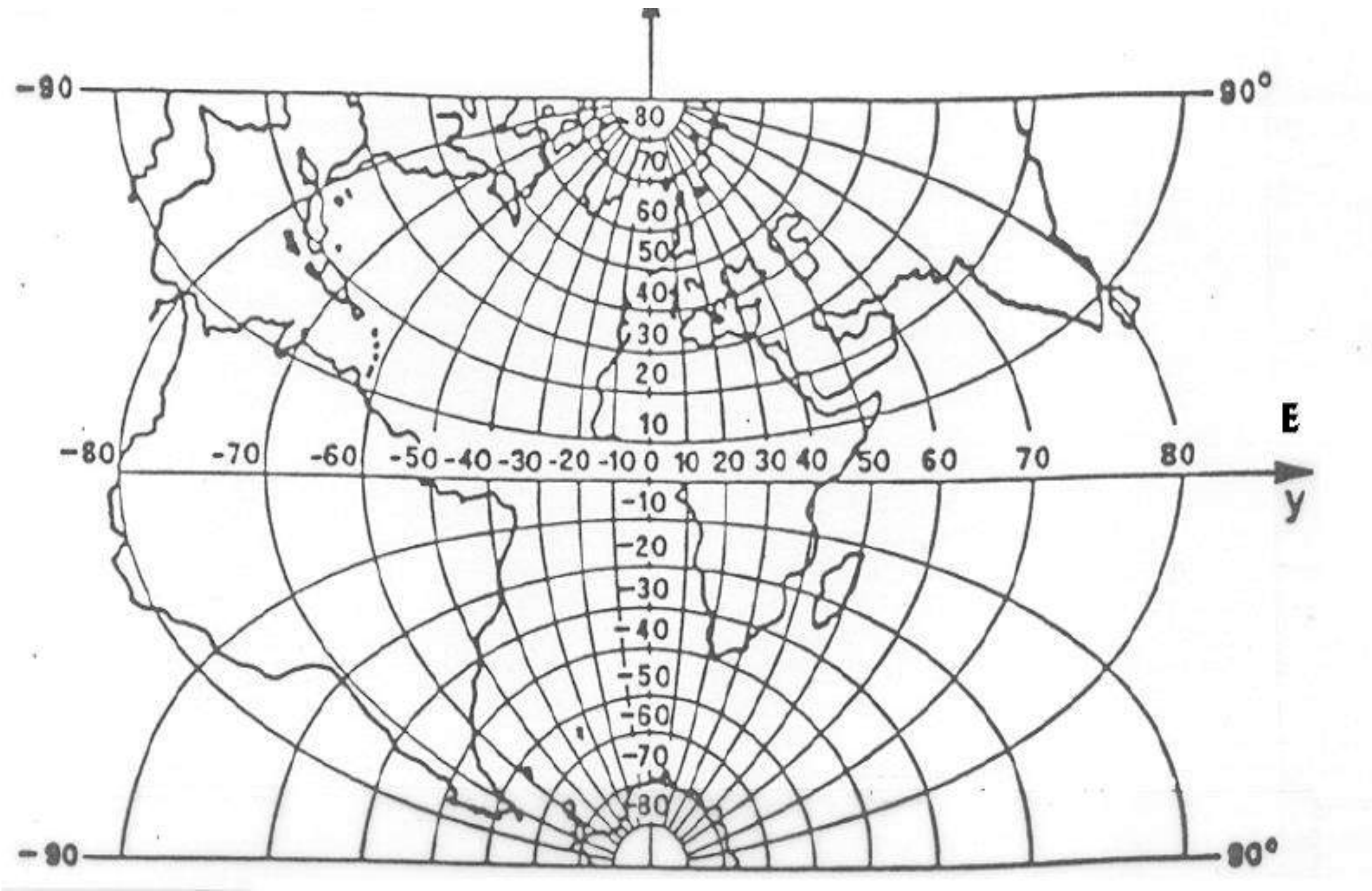


Fig. 8 — Il fuso di 6°.

Cylindrical projections

It derives from the cylindrical inverse

The central meridian and the equator are transformed into lines
The transformations of meridians and parallels are orthogonal to each other



Cylindrical projections

- If the map is conformal the deformation modulus ha to de constant with respect α.

$$\frac{dm^2}{d\alpha} = 0 \qquad m^2 = e^* \cdot \cos^2\alpha + 2f^* \cdot \text{sen}\alpha \cdot \cos\alpha + g^* \cdot \text{sen}^2\alpha$$

$$-2 \cdot e^* \cdot \cos\alpha \cdot \sin\alpha + 2 \cdot f^* \cdot \cos^2\alpha - 2 \cdot f^* \cdot \sin^2\alpha + 2 \cdot g^* \cdot \sin\alpha \cdot \cos\alpha = 0$$

$$2 \cdot f^* \cdot \cos(2\alpha) + (g^* - e^*) \cdot \sin(2\alpha) = 0$$

- After some mathematical tricks…

$$\begin{cases} \dfrac{\partial y}{\partial u} = -\dfrac{\partial x}{\partial \lambda} \\ \dfrac{\partial y}{\partial \lambda} = +\dfrac{\partial x}{\partial u} \end{cases}$$

Partial differential equations (Cauchy – Riemann Conditions)

- By using complex variables, Taylor series and other mathematical tricks

$$y = f(u) - \frac{1}{2} f^{II}(u)\lambda^2 + \frac{1}{4!} f^{IV}(u)\lambda^4 - \dots$$

$$x = f^{I}(u)\lambda - \frac{1}{3!} f^{III}(u)\lambda^3 + \frac{1}{5!} f^{V}(u)\lambda^5 - \dots$$

These conditions are valid for all conformal maps. The different maps differ in definition of the function f(u).

Cylindrical projections

- By imposing Gauss constraints (representation is equidistant on the central meridian)

$$y_{(\lambda=0)} = f(u) = \int_0^u r\,du = \int_0^\varphi \rho\,d\varphi$$

$$f'(u) = \frac{d}{du}\int_0^u r\,du = r = \frac{a\cdot\cos\varphi}{W} = N\cdot\cos\varphi$$

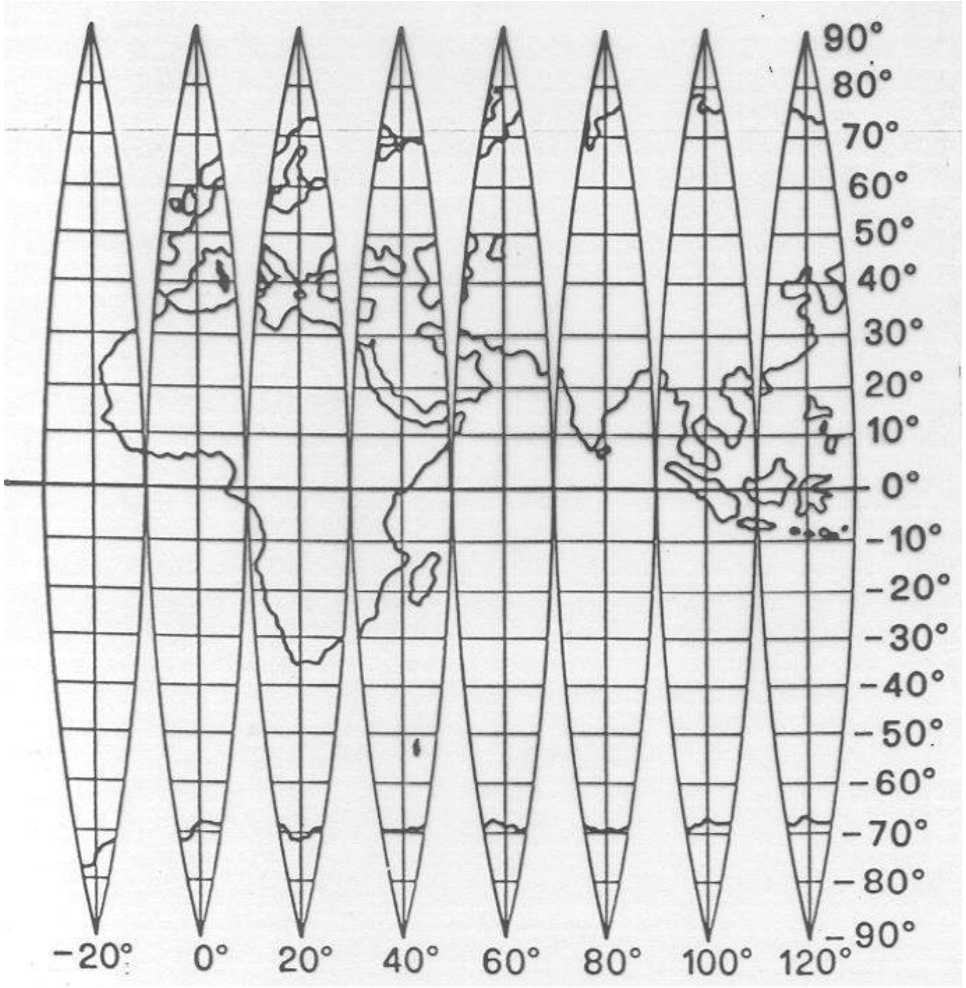
$$f''(u) = \frac{dr}{du} = \frac{dr}{d\varphi}\cdot\frac{d\varphi}{du} = -\rho\cdot \mathrm{sen}\,\varphi\cdot\frac{r}{\rho} = -r\cdot \mathrm{sen}\,\varphi = -N\cdot \mathrm{sen}\,\varphi\cdot\cos\varphi$$

- Again after some mathematical tricks we obtain the formula for the Gauss Map

$$\begin{cases} x = \lambda N\cos\varphi + \frac{1}{6}\lambda^3 N\cos^3\varphi(1-t^2+\eta^2) + \\ \quad + \frac{1}{120}\lambda^5 N\cos^5\varphi(5-18t^2+t^4+14\eta^2-58t^2\eta^2) \\ y = l_\varphi + \frac{1}{2}\lambda^2 N\,\mathrm{sen}\,\varphi\cos\varphi + \frac{1}{24}\lambda^4 N\,\mathrm{sen}\,\varphi\cos^3\varphi(5-t^2+9\eta^2+4\eta^4) \end{cases}$$

Gauss Map

To reduce the deformations on the map, a representation by spindles is used



Since the map is conformal the deformation module depends only on the position and can be calculated as:

$$m_{lineare} \cong 1 + \frac{x_A^2 + x_A x_B + x_B^2}{6\rho_{medio} N_{medio}}$$

UTM Projection

It derives from the Gauss compliant and is used for the representation of the whole terrestrial surface in areas with latitude between -80 ° and + 80 °

The ellipsoid used is WGS84

It is a representation for 6th-wide spindles, built around the central meridian

Each time zone is identified with a number (for Italy fused 32/33/34)

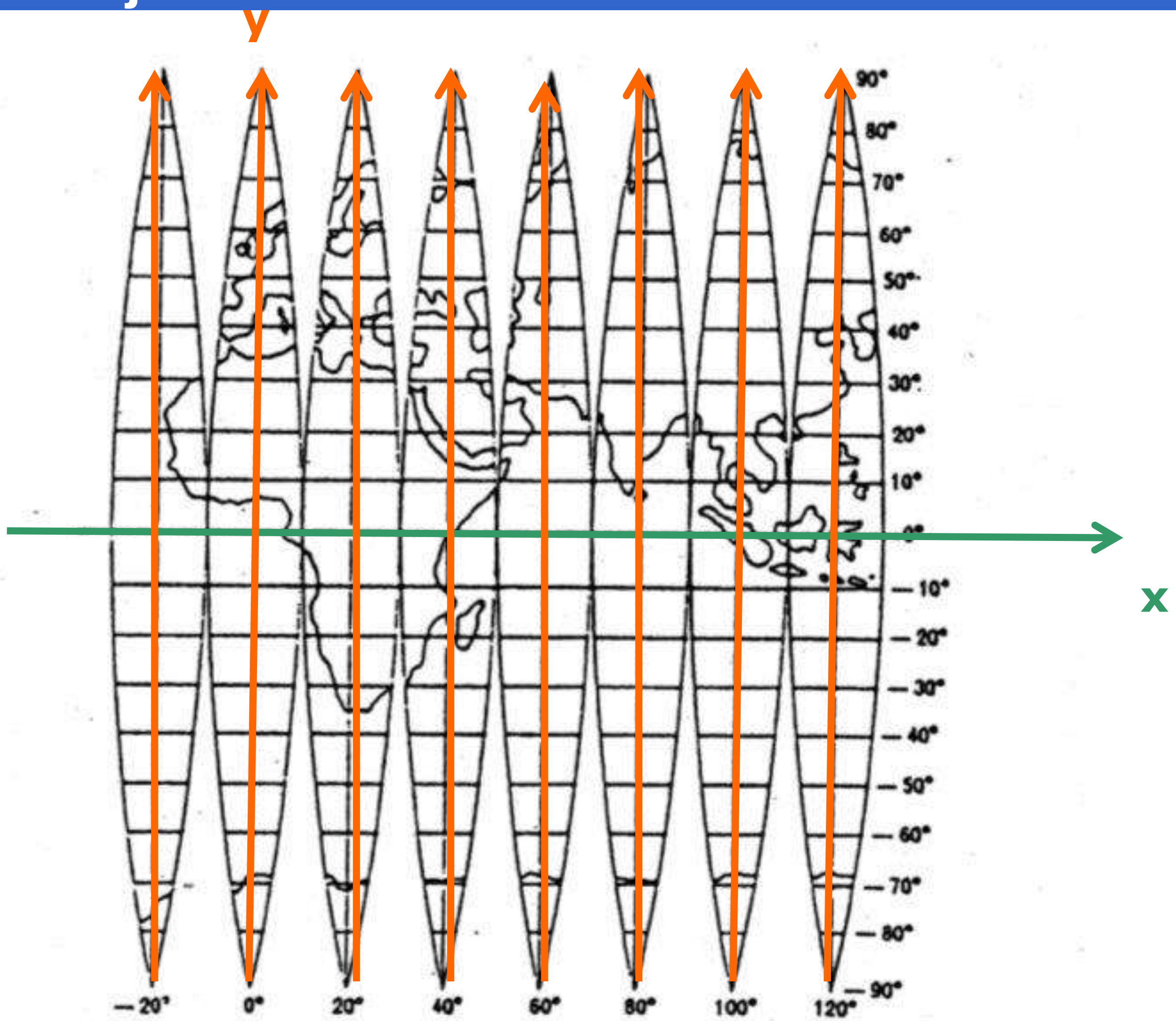
Each spindle has a Cartesian coordinate system where the North axis is represented by the central meridian of the spindle while the East axis is represented by the equator

At the east coordinates a false origin of 500 km is assigned

UTM Projection

- 60 spindles of 6 ° with 30 cylinders rotated 6 °
- Each spindle can be seen as independent, ie each spindle has its own X and Y axis
- I lose the uniqueness of the point in the cartographic plan because I have to specify in which spindle I'm working!
- E.g. WGS84 FUSO32
- Linear deformation module between 1 (tangent meridian) and 1,0008 (edge)
- I can therefore try to further reduce the differences between cartographic and ellipsoid planes

UTM Projection



UTM Projection

- Secant cylinder
- 0.9996 <m <1,0004
- Differences up to 40 cm per km, equal to 4/10000
- Values lower than the graphing (equal to 0.4 mm at the scale of the map)
- Central meridian of the spindle (not of tangency) - m = 0.9996
- Secrecy lines - m = 1.0000
- Edge - m = 1,0004

6°
12°
18°
48°
T
ROMA
M MARIO
40°
S
32°
32
33

Gauss-Boaga Map

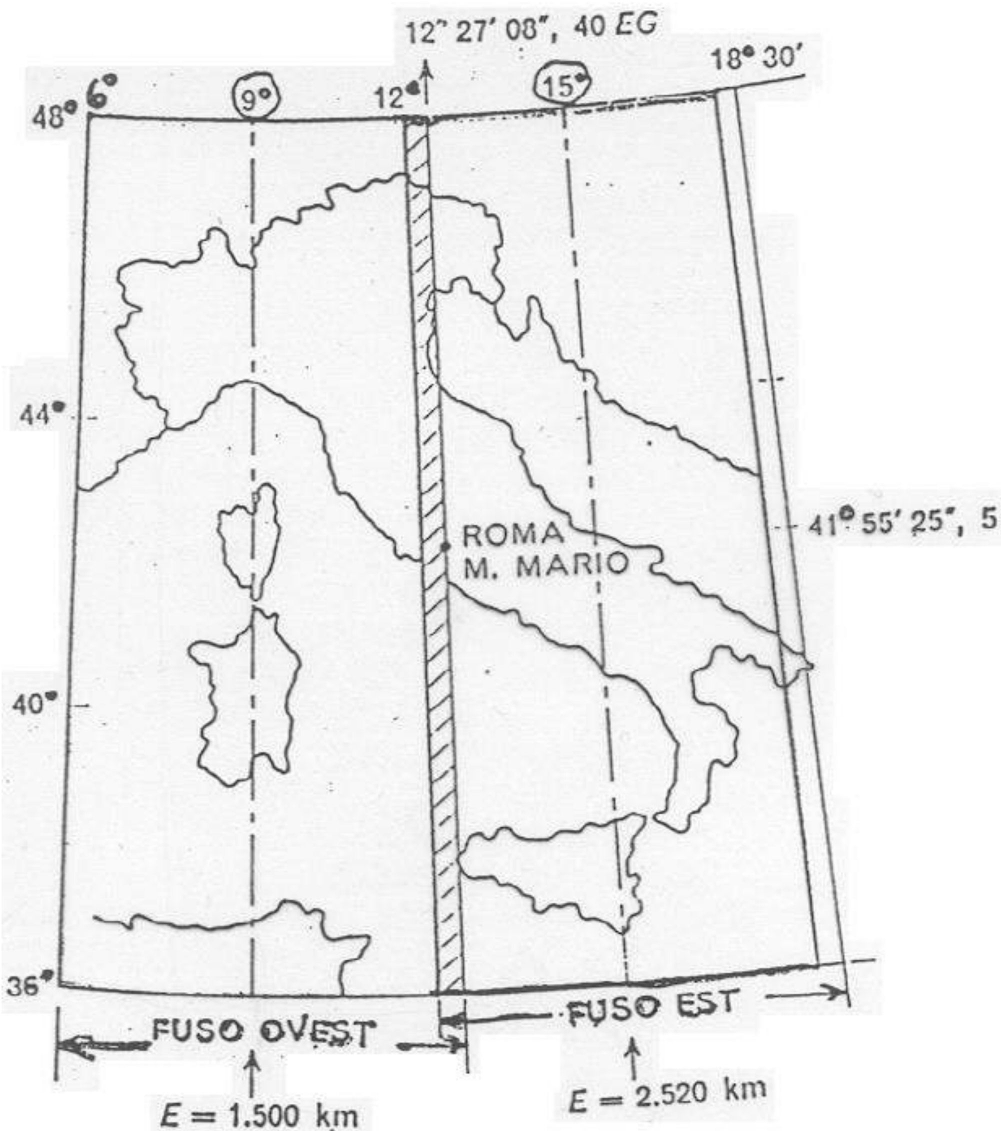
At the base of Italian cartography

It derives from the Gauss conformal with some modifications made by Giovanni Boaga

The ellipsoid used is the International oriented to Rome Monte Mario (1940)

Also in this case a representation for spindles is used in order to reduce the deformations

Gauss-Boaga Map



Italy is represented in two spheres (West and East) that have an overlapping area

The spindles have a false FO origin of 1500 km and 2520 km

Gauss-Boaga Map

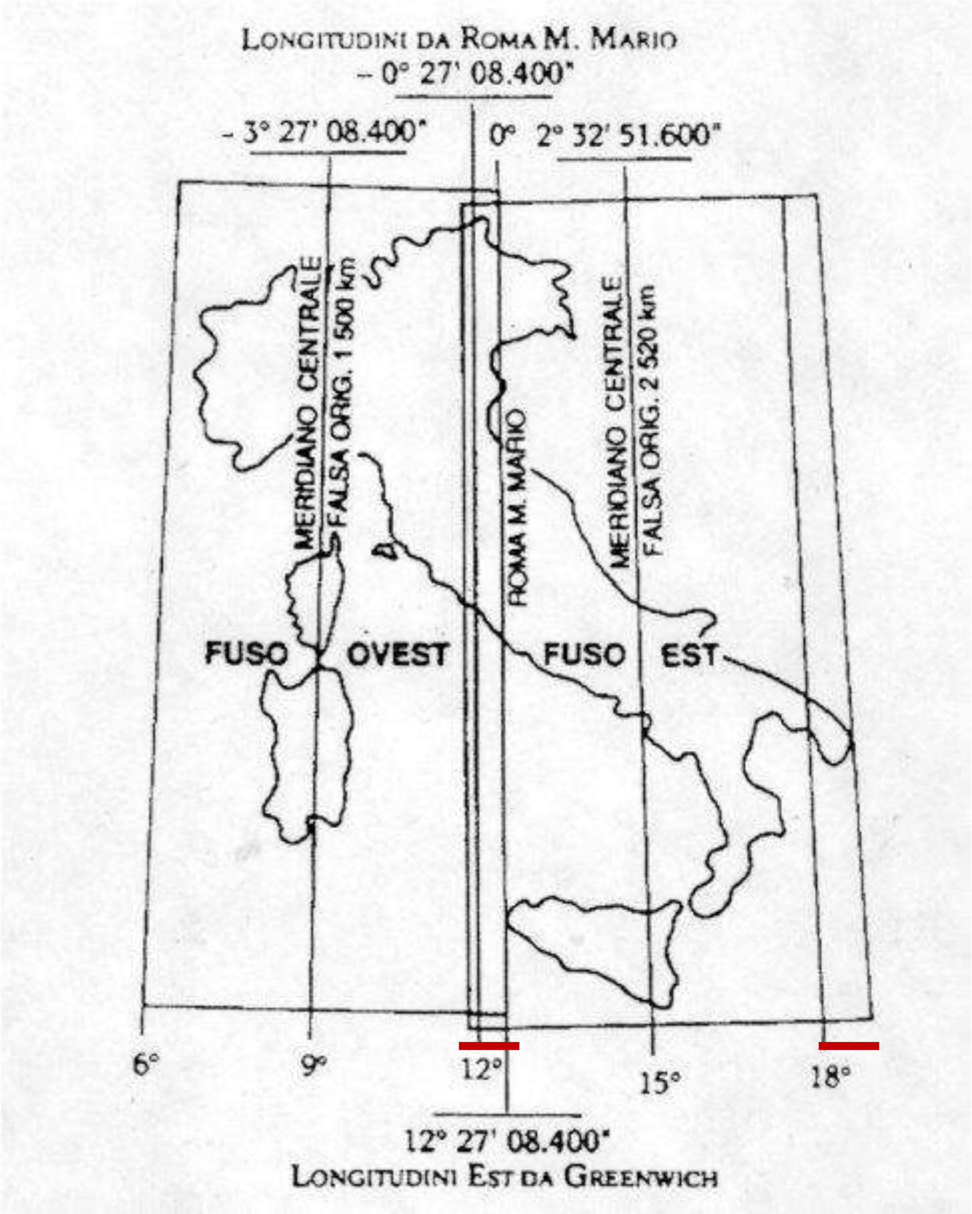
The Italian grid refers to the projection of Gauss-Boaga, an international ellipsoid, although in many Italian topographic maps at the 1: 25,000 scale both types of cross-linked (UTM and Gauss-Boaga) are indicated. Using the kilometric grid, the coordinates "x" and "y" of the point are given in meters.

- Italy is on 2 time zones: West and East
- Each time zone is 6 °
- The spindles are set from 6 ° to 12 ° (spindle West) and from 12 ° to 18 ° (spindle east)
- Each spindle is expanded to the east to generate a zone of overlap between the two spindles and to contain the Salento peninsula

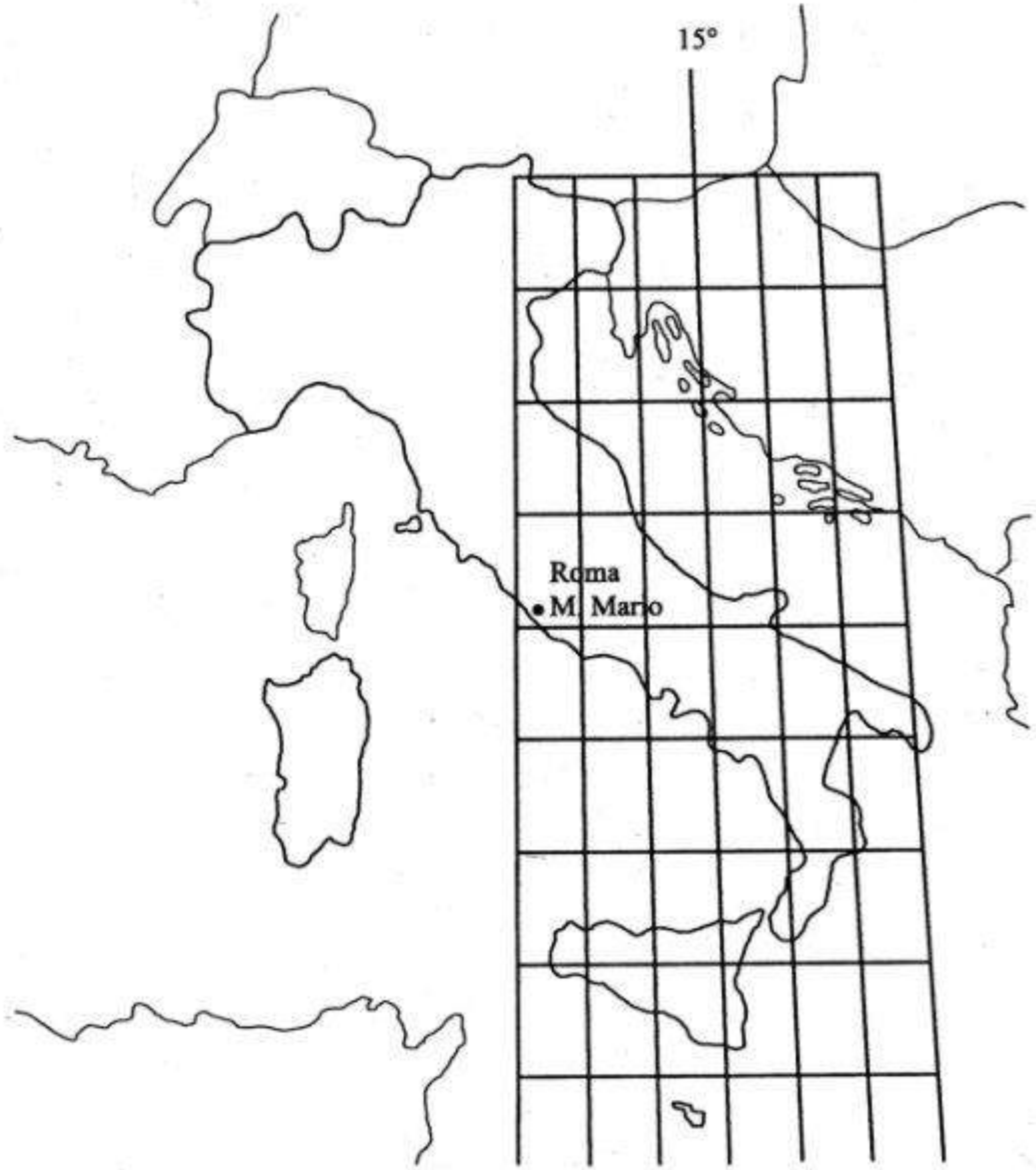
Consequently, the spindles go:

from 6 ° to 12 ° 30 '(West spindle) and from 12 ° to 18 ° 30' (East spindle)

Gauss-Boaga Map



Gauss-Boaga Map



Gauss-Boaga Map

- The national network compensated: DATUM Roma 40

- The value of the central meridian is equal to:
  - east = 1500 for the west zone
  - east = 2520 for the east zone
- If a point has coordinates Est = 1518 km

→ it is in the west zone (1) and is 18 km from the central meridian of the zone towards the east

- Reference for North is the equator

Name	*Identifier*	*CRS kind*	*CS Axes*
Monte Mario	**4265**	2D geographic	φ, λ
Monte Mario / Italy zone 1	**3003**	Projected	Nord, Est
Monte Mario / Italy zone 2	**3004**	Projected	Nord, Est

Name	*Identifier*	*CRS kind*	*CS Axes*
ED50	**4230**	2D geographic	φ, λ
ED50/UTM zone 32N	**23032**	Projected	Nord, Est
ED50/UTM zone 33N	**23033**	Projected	Nord, Est
ED50/UTM zone 34N	**23034**	Projected	Nord, Est

Name	*Identifier*	*CRS kind*	*CS Axes*
WGS 84	**4978**	geocentric	X,Y, Z
WGS 84	**4979**	3D geographic	φ, λ, h
WGS 84	**4326**	2D geographic	φ, λ
WGS 84 / UTM zone 32N	**32632**	Projected	Nord, Est
WGS 84 / UTM zone 33N	**32633**	Projected	Nord, Est
WGS 84 / UTM zone 34N	**32634**	Projected	Nord, Est

Convesrione between different reference systems